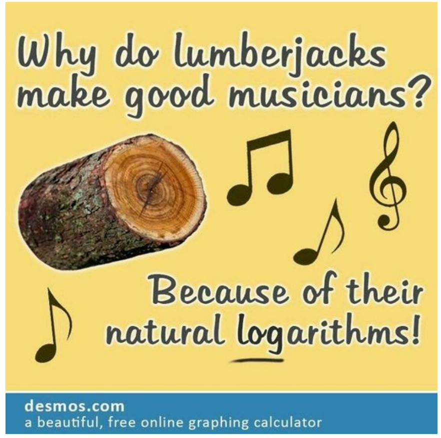
Precalculus

Lesson 5.4: Logarithmic Functions

Mrs. Snow, Instructor



The inverse of an exponential function is a logarithmic function.

Let a be a positive number with a not equal to 1. The logarithmic function with base a is defined by:

$$\boldsymbol{log_a x = y}$$

if and only if

$$\boldsymbol{a^y = x}$$

Domain: $(\mathbf{0}, \infty)$

translation: whatever you are taking the log of has to be greater than zero

Range: $(-\infty, \infty)$

start with the base and move in a counterclockwise fashion.

Change each logarithmic statement to an equivalent statement involving an exponent.

$log_a 4 = 5$	$log_e b = -3$	$log_3 5 = c$

Change each exponential statement to an equivalent statement involving a logarithm.

$1.2^3 = m$	$e^b = 9$	$a^4 = 24$

Find the exact value.

$log_2 16$	$log_3 \frac{1}{27}$

Find the domain of each logarithmic function.	
$f(x) = log_2(x + 3)$	$g(x) = log_5\left(\frac{1+x}{1-x}\right)$

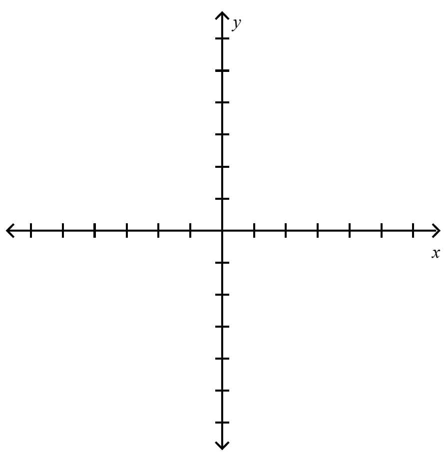
Graphing Logarithmic Functions

Knowing the general form of the graph of the log function is a short cut for graphing.

1. Write in its equivalent exponential form
2. Find the inverse; $x\ is\ y\ and\ y\ is\ x$, solve for y
3. Graph the log function's inverse, and reflect the exponential graph across the line of symmetry $y = x$.

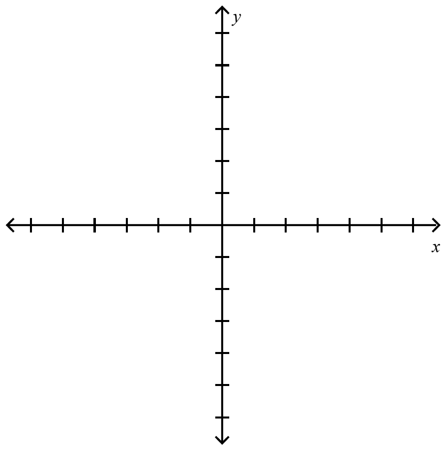
Graph, determine the domain, range and vertical asymptote.

$$y = log_2 x$$



Graph, determine the domain, range and vertical asymptote.

$$y = log_{1/3}x$$



The Natural and Common Logarithm

The Natural Logarithm is a logarithm with the base e. It is written with the abbreviation of ln.

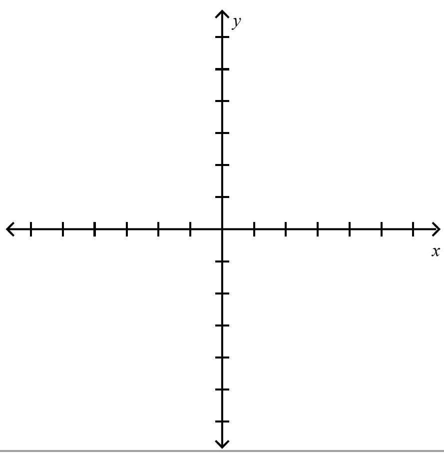
$$y = \ln x, \quad \text{if and only if} \quad x = e^y$$

$$y = \log x, \quad \text{if and only if } x = 10^y$$

The logarithm with base 10 is called the ***common logarithm*** *and is denoted by omitting the base*

Graph, determine the domain, range and vertical asymptote. Identify the inverse and the domain and range of the inverse.

$$f(x) = -\ln(x-2)$$



Solving Logarithmic Equations

$log_3(4x-7) = 2$	$log_x 64 = 2$

Using Logarithms to Solve and Exponential Equation

$$e^{2x} = 5$$

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Lesson 5.5: Properties of Logarithms

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Logarithms have some very useful properties that can be derived directly from the definition and the laws of exponents:

$$log_a 1 = 0$$

$$log_a a = 1$$

$$a^{log_a M} = M$$

$$log_a a^r = r$$

Simplify:

$2^{log_2 \pi}$	$log_{0.2} 0.2^{-\sqrt{2}}$
$ln e^{kt}$	$log_4 4$

Properties of Logarithms

$$log_a MN = log_a M + log_a N$$

$$log_a \frac{M}{N} = log_a M - log_a N$$

$$log_a M^r = r\, log_a M$$

Write the logarithmic expressions as Sum and Difference Logs

$log_a\left(x\sqrt{x^2+1}\right), x > 0$
$ln\frac{x^2}{(x-1)^3}$
$log_a\frac{\sqrt{x^2+1}}{x^3(x+1)^4}$

Writing Expressions as a Single Logarithm

$log_a 7 + 4log_a 3$	$\frac{2}{3}\ln 8 - \ln(5^2 - 1)$
$log_a x + log_a 9 + log_a(x^2 + 1) - log_a 5$	

Change of Base

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then $$\log_a M = \frac{\log_b M}{\log_b a}$$
Therefore: $$\log_a M = \frac{\log M}{\log a} \quad \text{and} \quad \log_a M = \frac{\ln M}{\ln a}$$

Using the Change of Base Formula

$log_5 89$	$log_{\sqrt{2}}\sqrt{5}$