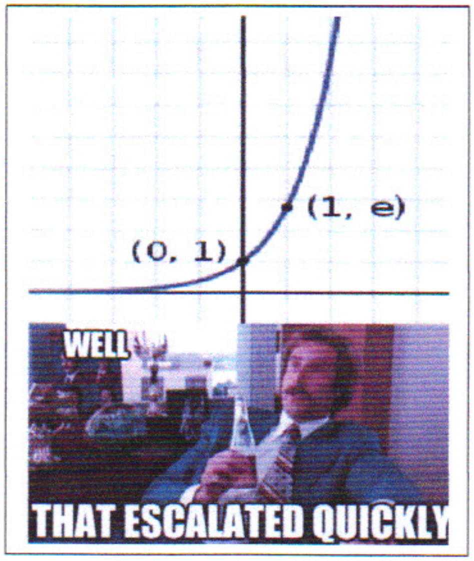
Calculus
Lesson 5.4 Exponential Functions: Differentiation and Integration
Mrs. Snow, Instructor



As we learned back in Algebra II, logarithm functions and exponential functions are inverses of one another. As Algebra II was a long time ago, let's do a quick review.

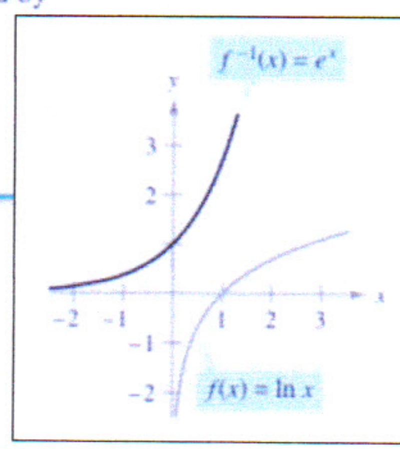
DEFINITION OF THE NATURAL EXPONENTIAL FUNCTION

The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

$$y = e^x \quad \text{if and only if} \quad x = \ln y.$$



$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$

Solving Exponential and Logarithmic Equations

$\ln 7 = \overset{\ln}{e^{x+1}}$

$\ln 7 = \ln e^{x+1}$

$\ln 7 = x+1 \cancel{\ln e}^{1}$

$-1 + \ln 7 = x \approx .946$

$\ln(2x-3) = 5$

$\log_e$

$e^5 = 2x - 3$

$\dfrac{e^5+3}{2} = x \approx 75.707$

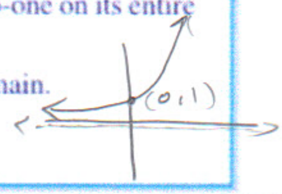
THEOREM 5.10 OPERATIONS WITH EXPONENTIAL FUNCTIONS

Let a and b be any real numbers.

1. $e^a e^b = e^{a+b}$
2. $\frac{e^a}{e^b} = e^{a-b}$

PROPERTIES OF THE NATURAL EXPONENTIAL FUNCTION

1. The domain of $f(x) = e^x$ is $(-\infty, \infty)$, and the range is $(0, \infty)$.
2. The function $f(x) = e^x$ is continuous, increasing, and one-to-one on its entire domain.
3. The graph of $f(x) = e^x$ is concave upward on its entire domain.
4. $\lim_{x \to -\infty} e^x = 0$ and $\lim_{x \to \infty} e^x = \infty$



The natural exponential function is its own derivative!!

THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION

Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$
2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

a. $\frac{d}{dx}[e^{2x-1}] = e^u \frac{du}{dx}$

$(e^{2x-1})(2) = 2e^{2x-1}$

b. $\frac{d}{dx}[e^{-3/x}] = e^u \frac{du}{dx} = e^{-3/x}\left(\frac{3}{x^2}\right)$

$= \frac{3e^{-3/x}}{x^2}$

$\frac{d}{dx} -\frac{3}{x} = -3x^{-1}$

$3x^{-2}$

Locating Relative Extrema

f(x) = xe^x

$f'(x) = (1)(e^x) + (e^x)(x)$

$= e^x(1+x) = 0$

$\ln e^x = \ln 0$

$\cancel{x = \ln 0}$

$1 + x = 0$

$x = -1$

$-2 \quad 0$

$- \quad -1 \quad +$

$\searrow \quad \nearrow$

$f(-1) = (-1)(e^{-1})$

$\min\left(-1, \frac{-1}{e}\right)$

THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x\,dx = e^x + C$ 2. $\int e^u\,du = e^u + C$

u-Substitution

Integrating Exponential Functions

Find $\int e^{3x+1}\,dx$.

$u = 3x+1$
$du = 3\,dx$
$\frac{1}{3}du = dx$

$= \frac{1}{3}\int e^u\,du$

$= \frac{1}{3}e^u =$

$= \frac{1}{3}e^{3x+1} + C$

Find $\int 5xe^{-x^2}\,dx$.

$u = -x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$= \frac{-5}{2}\int e^u\,du$

$= \frac{-5}{2}e^u$

$= \frac{-5}{2}e^{-x^2} + C$

Integrating Exponential Functions

a. $\int \frac{e^{1/x}}{x^2}\,dx = \int e^{1/x}\left(\frac{1}{x^2}\right)dx$

$u = \frac{1}{x} = x^{-1}$
$du = -\frac{1}{x^2}dx$
$-du = \frac{1}{x^2}dx$

$= -\int e^u\,du$

$= -e^u$

$= -e^{1/x} + C$

b. $\int \sin x\, e^{\cos x}\,dx =$

$u = \cos x$
$du = -\sin x\,dx$
$-u = \sin x\,dx$

$-\int e^u\,du =$

$-e^u = -e^{\cos x} + C$

Finding Areas Bounded By Exponential Functions

a. $\int_0^1 e^{-x}\,dx =$

$-e^{-x}\Big|_0^1 =$

$-e^{-1} - (-e^0)$

$-\frac{1}{e} + 1$

b. $\int_0^1 \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$
$du = e^x\,dx$

$\int_0^1 \frac{1}{u}\,du = \ln|u|$

$= \ln|1+e^x|\Big|_0^1 =$

$= \ln|1+e^1| - \ln|1+e^0|$

$= \ln|1+e| - \ln|2|$

c. $\int_{-1}^0 [e^x\cos(e^x)]\,dx$

$u = e^x$
$du = e^x\,dx$

$\int_{-1}^0 \cos u\,du =$

$\sin u = \sin e^x\Big|_{-1}^0$

$= \sin e^0 - \sin e^{-1}$

$\boxed{= \sin 1 - \sin\frac{1}{e}}$

$\approx .482$