Precalculus
Lesson 8.2: Law of Sines
Mrs. Snow, Instructor

Until now, our work with triangles and trig functions has been limited to right triangles. However, what we have learned to date in this class allows us to work with all triangles. A triangle has three sides and three angles. If we know any three of these six measurements with one of the known measurements being a side, the other three measures may be found.

A triangle that does not contain a right angle is known as an **oblique triangle.** The data required to solve an oblique triangle may be listed in four cases. The first two cases are studied in this section and the other two will be studied in the next section.

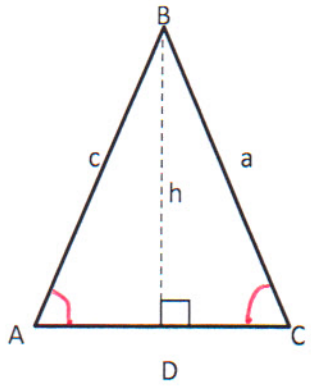
Case 1 – One side and two angles are known (SAA or ASA).

Case 2 – Two sides and one angle not included between the two sides are known (SSA). This case may lead to more than one triangle.

Case 3 – Two sides and the included angle are known (SAS).
Case 4 – Three sides are known (SSS).

LAW OF SINES

The Law of Sines may be derived either by using an acute or obtuse triangle.



While ΔABC is acute if we drop a perpendicular we get right triangles, ΔABD and ΔBCD. Using our trig functions we find:

$$\sin A = \frac{h}{c} \quad or \quad h = c \sin A$$

$$\sin C = \frac{h}{a} \quad or \quad h = a \sin C$$

setting equal to each other:

$$a \sin C = c \sin A \quad or \quad \frac{a}{\sin A} = \frac{c}{\sin C}$$

Similarly, you may find a relationship for angle B and then you will have the final form of the Law of Sines.

LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or using reciprocals

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Translation: according to the Law of Sines, the ratio of the length of a side of a triangle to the sine of the angle opposite the side is proportionate to the other ratios of sides to sine of angle opposite to the side. (I think I will stick to the formula and understand that!)

Strategy: When using the Law of Sines, select an equation so that the unknown variable is in the numerator and all other variables are known. ***DRAW THE TRIANGLE AND CAREFULLY LABEL!!!!***

SAA

Solve the triangle:

$A = 40°, B = 60°, a = 4$

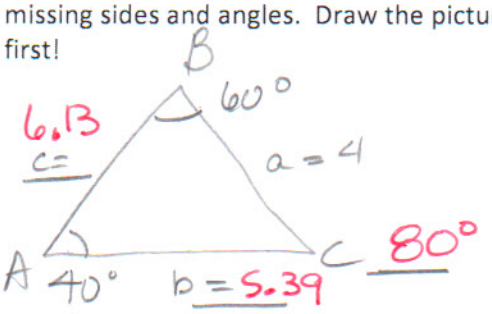
$\frac{4}{\sin 40} = \frac{c}{\sin 80}$

$c = \frac{4}{\sin 40}(\sin 80) = 6.13$

$\frac{4}{\sin 40} = \frac{b}{\sin 60}$

$b = \frac{4}{\sin 40}(\sin 60) = 5.39$

Solve the triangle (translation) find the missing sides and angles. Draw the picture first!



$180 - 100 = 80°$

ASA

Solve the triangle:

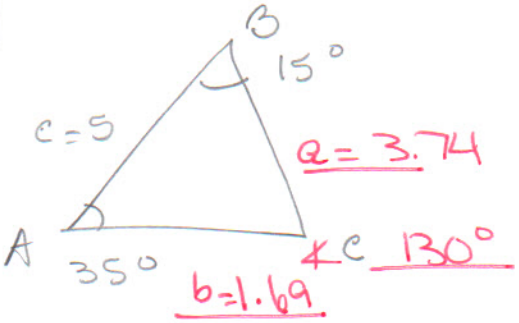
$A = 35°, B = 15°, c = 5$

$\frac{5}{\sin 130} = \frac{a}{\sin 35°}$

$\sin 35\left(\frac{5}{\sin 130}\right) = a = 3.74$

$\frac{5}{\sin 130} = \frac{b}{\sin 15°}$

$\frac{5}{\sin 130}(\sin 15°) = 1.69 = b$



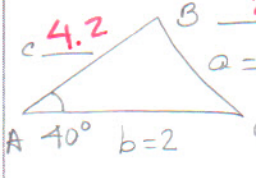
$180 - 50 = 130°$

Can there be more than one $\angle B$? (why would I ask this question if the answer were no??) This is what we have in Case 2, SSA. Case 2 is referred to as the **ambiguous case.** The information given may result in one triangle, two triangles or perhaps no triangle at all. Check out your textbook, page 523, for examples demonstrating these possible situations.

SSA– one solution.
Solve triangle:

$\angle A = 40°, a = 3 \text{ and } b = 2$

Picture is step #1!



$\angle C = 180 - 40 - 25.4 = 114.6° = C$

$\frac{\sin 40}{3} = \frac{\sin B}{2}$

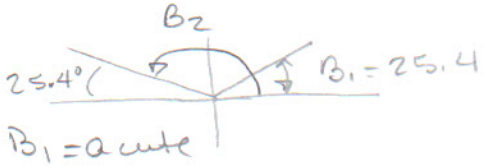
$\sin B \sim .428\ldots$

$B = 25.4°$

$\frac{3}{\sin 40} = \frac{c}{\sin 114.6}$ $c = 4.2$

Δ #2 ??

there are 2 possible $\angle B$



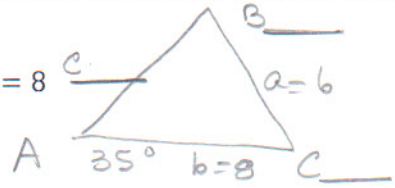
B_1 = acute

B_2 = obtuse = 180 - 25.4 = 154.6°

IF $\angle B = 154.6°$ & given $\angle A = 40°$ } 194.6°

What is $\angle C$? 180 − 194.6 } Not Possible

SSA – two solutions
Solve triangle:
$\angle A = 35°, a = 6, \text{ and } b = 8$
picture???



$\frac{\sin 35}{6} = \frac{\sin B}{8}$

$\sin B = .764\ldots$

$B = 49.9°$

$\frac{6}{\sin 35} = \frac{c}{\sin 95.1}$

$c = 10.4$

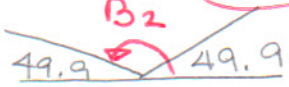
Δ #1

$\angle A = 35$ $a = 6$

$\angle B = 49.9°$ $b = 8$

$\angle C = 95.1°$ $c = 10.4$

Δ #2 ?? yes



$180 - 49.9$

$B_2 = 130.1$

$A = 35°$

$\angle C = 180 - 130.1 - 35$

$= 14.9°$

Δ #2

$\angle A = 35$ $a = 6$

$\angle B = 130.1$ $b = 8$

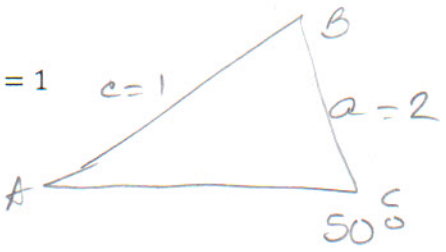
$\angle C = 14.9$ $c = 2.7$

$\frac{c}{\sin 14.9} = \frac{6}{\sin 35}$

SSA – no solution

Solve triangle:

$\angle C = 50°\ a = 2\ and\ c = 1$



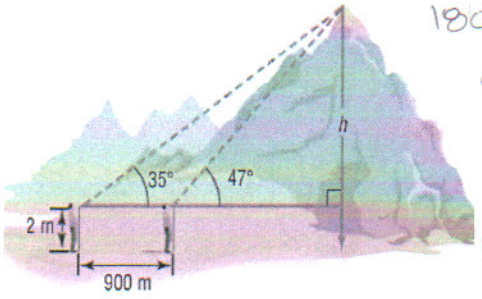
$\frac{\sin A}{2} = \frac{\sin 50}{1}$

$\sin A = 1.53...$ (Max is 1)

$A = \sin^{-1} 1.53...$

No sol

To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance of 900 meters apart on a direct line to the mountain. The first observation results in an angle of elevation of 47°, and the second results in an angle of elevation of 35°. If the transit is 2 meters high, what is the height, h, of the mountain?



180-90-47 =

180-90-35 = 55°

43°

55-43

12°

h

b

35°

47°

900

2m

+2m

$\frac{900}{\sin 12} = \frac{b}{\sin 35}$

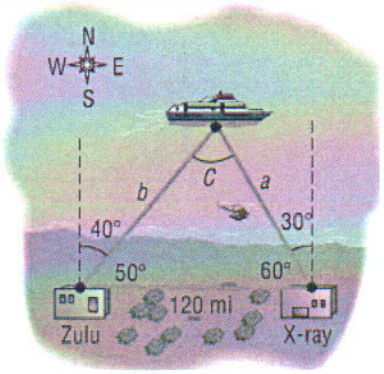
$b = 2482.88'$

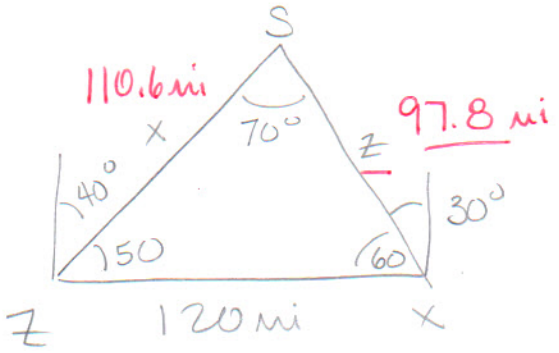
$\sin 47° = \frac{h}{2482.88}$

$h \approx 1816$ ft

$+2m =$ 1818ft

Coast Guard Station Zulu is located 120 miles due west of Station X-ray. A ship at sea sends an SOS call that is received by each station. The call to Station Zulu indicates that the bearing of the ship from Zulu is $N40°E$. The call to Station X-ray indicates that the bearing of the ship from X-ray is $N30°W$. How far is each station from the ship? If a helicopter capable of flying 200 mph is dispatched from the nearer station to the ship how long will it take to reach the ship?





$$\frac{Z}{\sin 50} = \frac{120}{\sin 70}$$

$$Z = 97.8 \text{ mi}$$

$$\frac{X}{\sin 60} = \frac{120}{\sin 70}$$

$$X = 110.6 \text{ mi}$$

Zulu to ship = 110.6 mi

X-Ray to ship = 97.8 mi

$$\frac{\text{hr}}{200 \text{ mi}}(97.8 \text{ mi}) = .489 \text{ hr}\left(\frac{60 \text{ mi}}{\text{hr}}\right) = 29.34 \text{ min}$$

Helicopter flight time