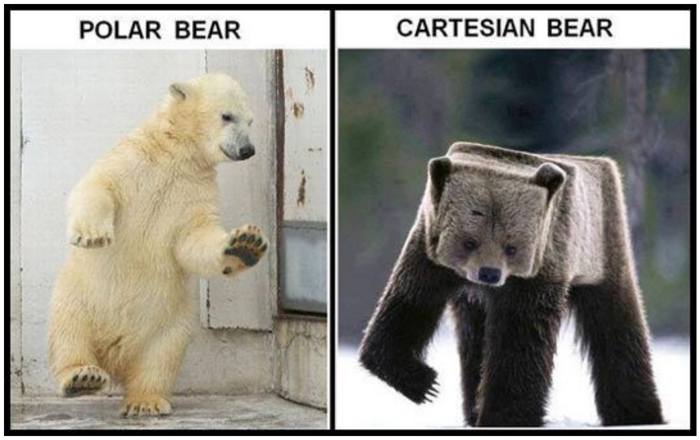
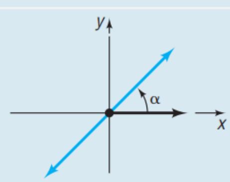
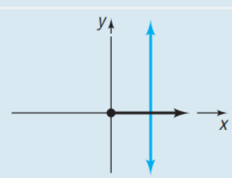
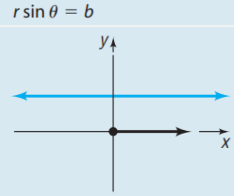
Precalculus
Lesson 9.2 Graphs of Polar Equations
Mrs. Snow, Instructor



As we studied last section points may be described in polar form or rectangular form. Likewise an equation may be written using either polar or rectangular coordinates. Depending on specific equation, one form may be easier to understand and graph than the other. Below are some common polar graphs and their equations written in both polar and rectangular forms.

Lines			
Description	Line passing through the pole making an angle α with the polar axis	Vertical line	Horizontal line
Rectangular equation	$y = (\tan \alpha)x$	$x = a$	$y = b$
Polar equation	$\theta = \alpha$	$r \cos \theta = a$	$r \sin \theta = b$
Typical graph			

To plot points we use polar coordinates and a polar grid.

The points (r, θ) *of*

$radius = 6, \quad with\ \theta = \frac{5\pi}{6}$ *and*

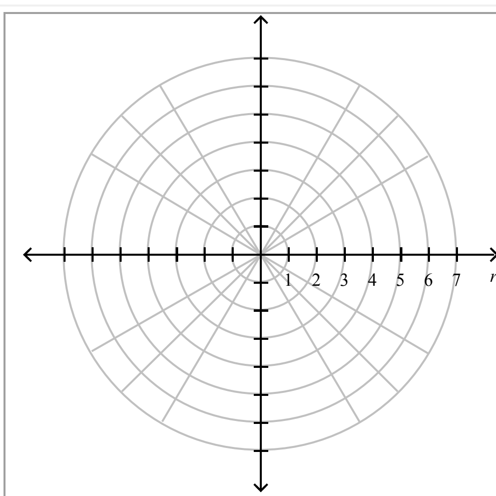
$radius = 4, \quad with\ \theta = \frac{\pi}{4}$

Special graphs:
θ = constant –a line at angle θ

r=constant –a circle of radius r

Sketch the graph of the equation and express the equation in rectangular coordinates:

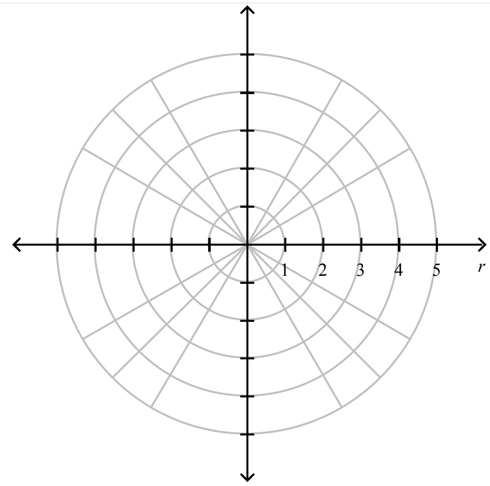
$r = 3$



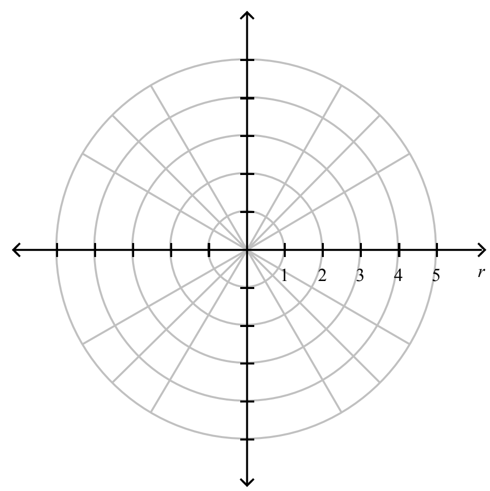
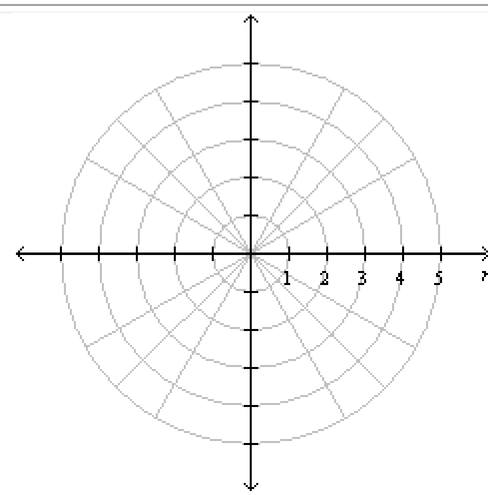
$\theta = \frac{\pi}{3}$

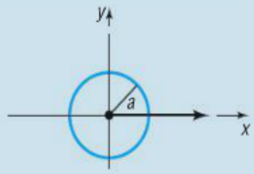
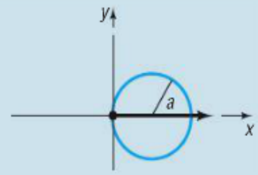
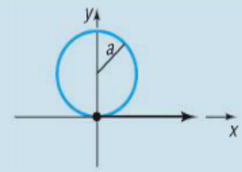
Graphing a Polar Equation of a Line:

Some equations can easily be expressed in rectangular coordinates. If this is the case then convert to rectangular coordinates.

Identify and graph the equation $\theta = \frac{\pi}{4}$	

Remember the formulas from section 1 that relate $x\ and\ y\ to\ r\ and\ \theta$:

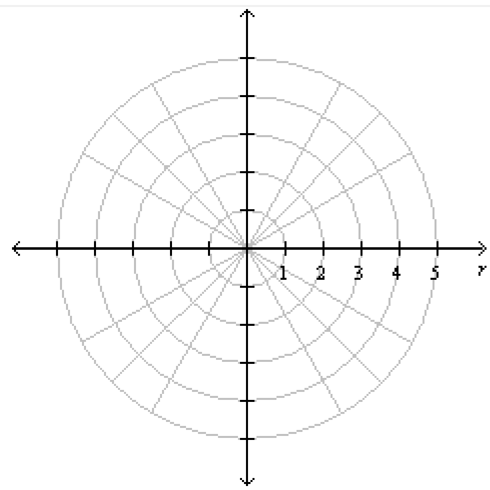
Identify and graph the equation $r \sin\theta = 2$	
Identify and graph the equation $r \cos\theta = -3$	

Circles			
Description	Center at the pole, radius a	Passing through the pole, tangent to the line $\theta = \frac{\pi}{2}$, center on the polar axis, radius a	Passing through the pole, tangent to the polar axis, center on the line $\theta = \frac{\pi}{2}$, radius a
Rectangular equation	$x^2 + y^2 = a^2, \quad a > 0$	$x^2 + y^2 = \pm 2ax, \quad a > 0$	$x^2 + y^2 = \pm 2ay, \quad a > 0$
Polar equation	$r = a, \quad a > 0$	$r = \pm 2a\cos\theta, \quad a > 0$	$r = \pm 2a\sin\theta, \quad a > 0$
Typical graph			

Graphing a Circle

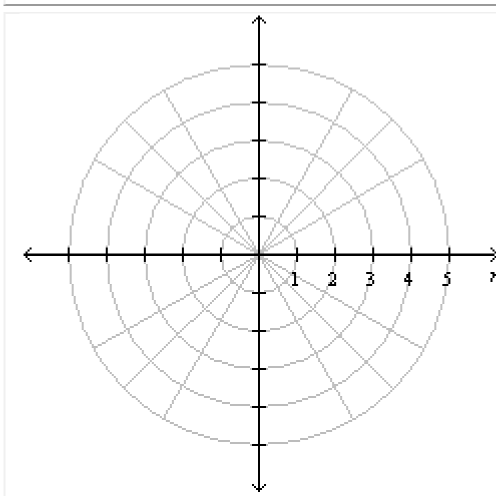
Sketch the polar equation (transform the equation into its rectangular form)

$$r = 4\sin\theta$$



Sketch the polar equation

$$r = -2\cos\theta$$



Other Equations (pg. 581)

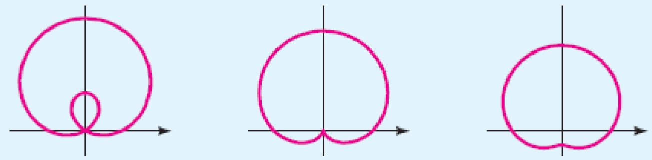
Limaçons

$r = a \pm b \sin\theta$

$r = a \pm b \cos\theta$

$(a > 0, b > 0)$

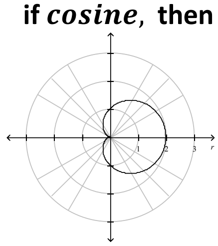
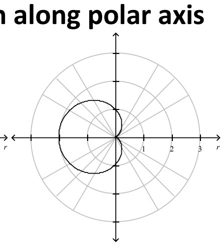
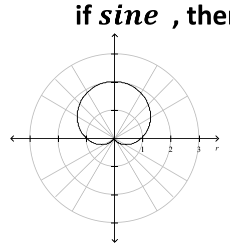
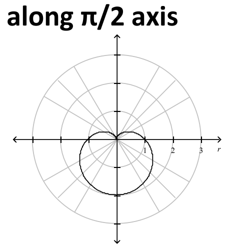
Orientation depends on the trigonometric function (sine or cosine) and the sign of b.



$a < b$ limaçon with inner loop | $a = b$ cardioid | $a > b$ dimpled limaçon

Name	Limaçon inner loop	Cardioid	Limaçon **no** inner loop has a dimple
Polar Equation	$r = a \pm \mathrm{b}\cos\theta$ $r = a \pm \mathrm{b}\sin\theta$ $a < b$	$r = a \pm \mathrm{a}\cos\theta$ $r = a \pm \mathrm{a}\sin\theta$ $a = b$	$r = a \pm \mathrm{b}\cos\theta$ $r = a \pm \mathrm{b}\sin\theta$ $a > b$

Equations in terms of cosine will be symmetrical about the polar axis (horizontal).
Equations in terms of sine will be symmetrical about the $\frac{\pi}{2}$ axis (vertical).

Cardioid graphs	
$\underline{a = b, \quad distance\ on\ axis\ is\ 2a}$ **if $\boldsymbol{cosine}$, then along polar axis**   $r = 1 + \cos\theta$ $\quad$ $r = 1 - \cos\theta$	**if $\boldsymbol{sine}$, then along π/2 axis**   $r = 1 + \sin\theta$ $\quad$ $r = 1 - \sin\theta$

Cardioid – heart shaped (pg. 581)

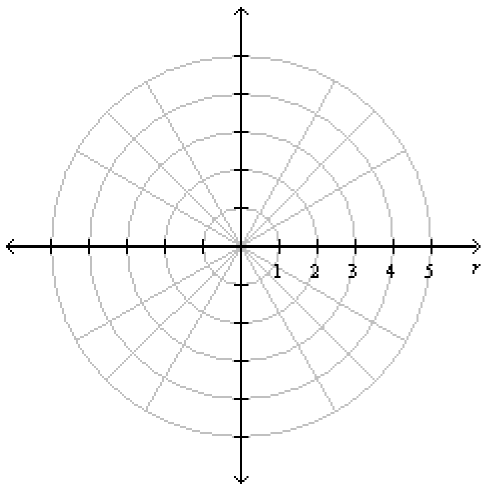
graph $r = 2 - 2\sin\theta$

a=__________ b=_________

The numbers indicate a shape of _______________

equation has sine so along __________________axis

Negative means: length =_________



Whenever you cannot remember how to graph the polar equation, you can always graph a period of the trig function from $0 \le \theta < 2\pi$ *and transfer the data over to a polar graph. Don't rely on memorizing an equation and associated graph shape, you will want a backup method!!*

Table of values *(use values for theta that yield friendly values for r):*

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	1/2	1	1/2	0	-1	0
$r = 2 - 2\sin\theta$	2	1	0	1	2	4	2

Limaçon graphs

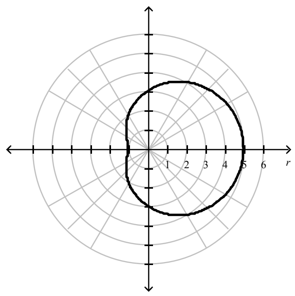
$$r = a \pm \mathbf{b}\cos\theta$$
$$r = a \pm \mathbf{b}\sin\theta$$

if $cosine$: along polar axis

if $sine$: along $\frac{\pi}{2}$ axis

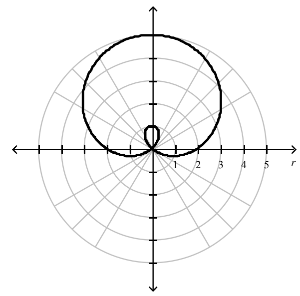
a. Limacon no inner loop if :

$a > b$



b. Limacon has an inner loop if:

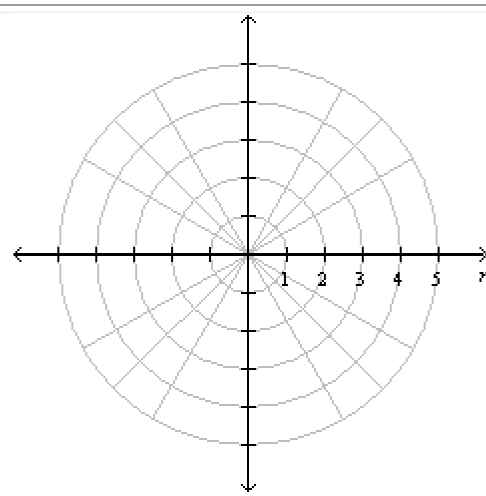
$a < b$



Graphing a limaçon without an inner loop

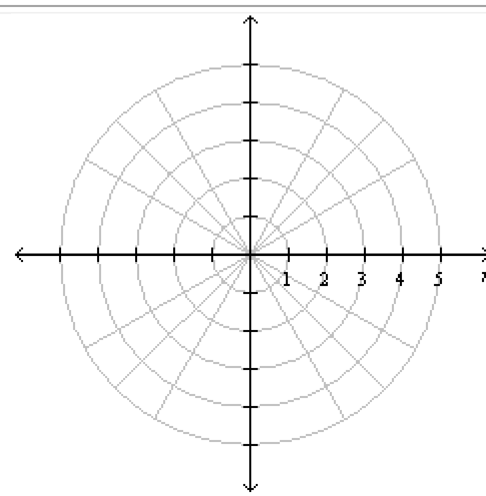
Sketch the graph of the equation

$r = 3 + 2cos\,\theta$



Graphing a limaçon with an inner loop

$r = 1 + 2\,cos\,\theta$



More Equations

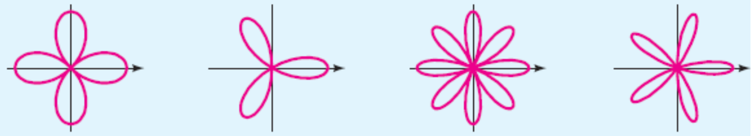
Roses

$r = a \sin n\theta$

$r = a \cos n\theta$

n-leaved if n is odd

$2n$-leaved if n is even



$r = a\cos 2\theta$ 4-leaved rose | $r = a\cos 3\theta$ 3-leaved rose | $r = a\cos 4\theta$ 8-leaved rose | $r = a\cos 5\theta$ 5-leaved rose

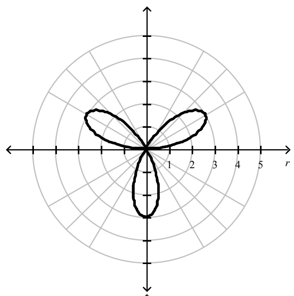
Rose with even/odd petals

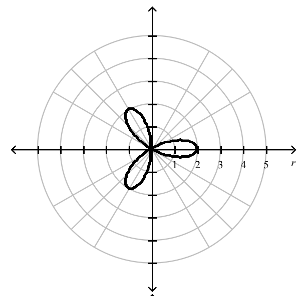
$$n: \begin{cases} odd = n \ \ petals \\ even = 2n \ \ petals \end{cases}$$

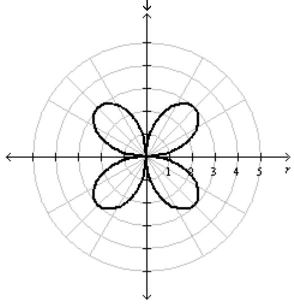
$$a = length\ of\ petal$$

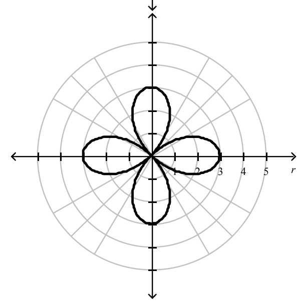
$$\boldsymbol{r = a\sin n\theta}$$

$$\boldsymbol{r = a\cos n\theta}$$



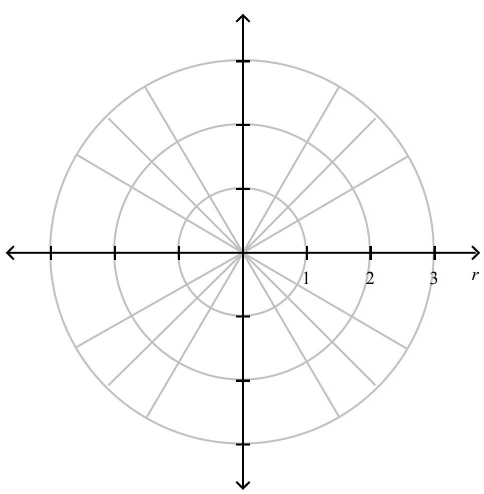




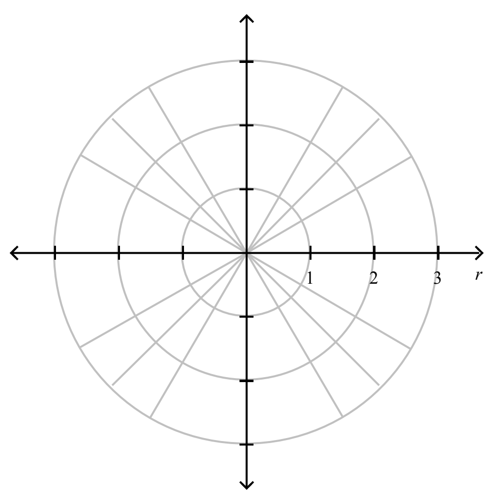


Graphing a Polar Equation: n-leaved rose (petals)

$r = 2\sin 3\theta$

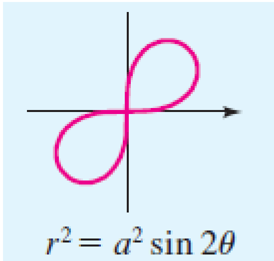


$r = 2\cos 2\theta$

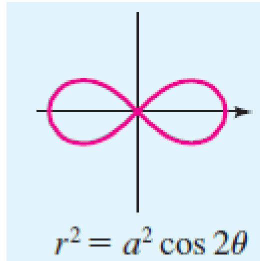


Lemniscates – Figure 8 shaped curves

$r^2 = a^2 sin2\theta$

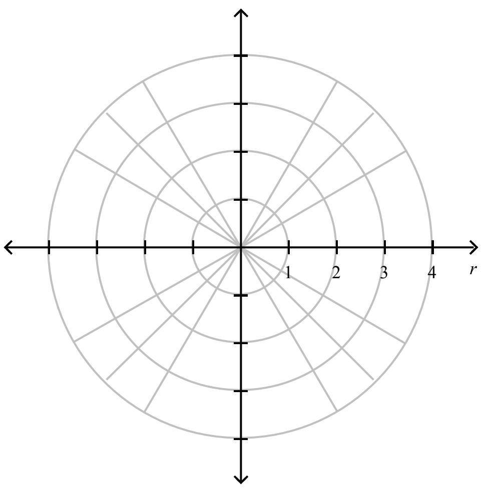


$r^2 = a^2 cos2\,\theta$

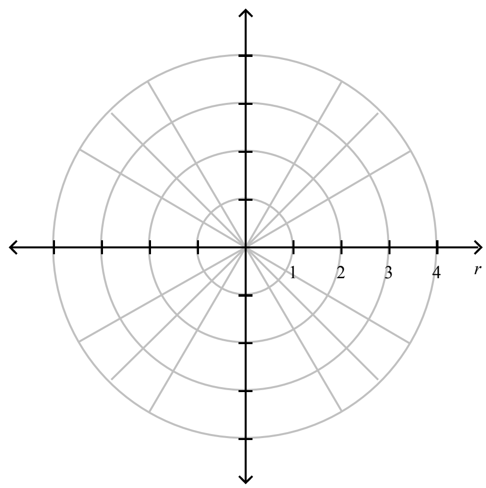


$a = petal\ length$

$r^2 = 9sin2\theta$



$r^2 = 2^2 cos2\theta$



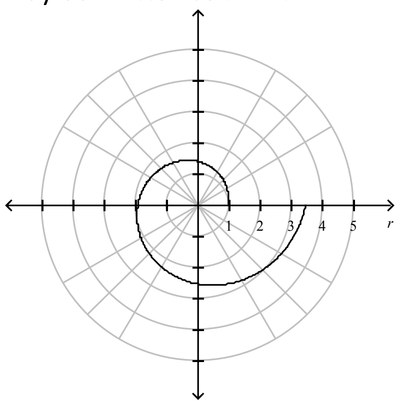
Graphing a Polar Equation (spiral)

It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

There are several equations that will produce a spiral. The **logarithmic spiral**

$$r = e^{\theta/5}$$

may be written as $\theta = 5\ln r$



Archimedes Spiral is in the form of

$$r = a\theta$$