Precalculus
Lesson 9.1: Polar Coordinates
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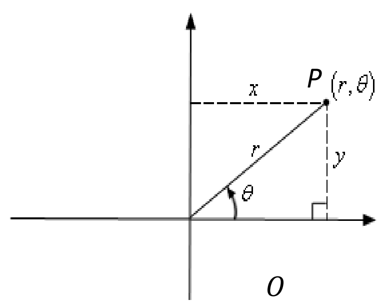
Coordinate systems are nothing more than a way to describe a point in space. The rectangular or Cartesian coordinates describe a point on a grid system moving from the origin *x* units horizontally followed by *y* units vertically. There is another way to describe a point in dimensional space. Polar coordinates describe the location of a point as a distance ***r*** from the origin and an angle θ determined from the positive x-axis. Polar coordinates are useful when working with more complicated equations such as those for a circle, ellipse, or a figure 8.



Polar coordinate system: uses distances and directions to specify the location of a point in the plane.
polar axis: the horizontal axis.
polar coordinates: $\boldsymbol{r}$ is the distance from the origin (pole) to the point **P.** $\boldsymbol{\theta}$ is the angle formed by the terminal side (it is between the polar axis and the segment $\overline{\boldsymbol{OP}}$**).**
The terminal side defines the positive direction of the distance away from the pole.
Point **P** is described by the ordered pair $(r, \theta) \rightarrow (distance, angle)$

Note:

- In polar coordinates there are literally an infinite number of coordinates for a given point.
- θ is considered positive if measured in a counterclockwise direction from the polar axis or negative if measured in a clockwise direction.
- If r is negative, then $P\ (-r, \theta)$ is defined to be the point that lies $|r|$ units from the pole in the direction <u>opposite</u> to that given by θ. The negative sign in front of *r* is directional.
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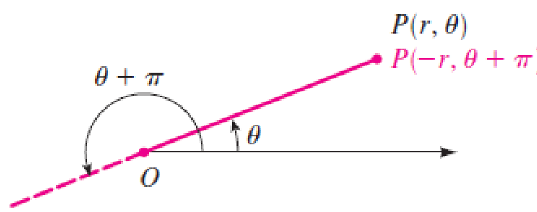
EXAMPLE – Plot the polar coordinates

$\left(3, \frac{5\pi}{3}\right)$	$\left(2, -\frac{\pi}{4}\right)$	$(3, 0)$	$\left(-2, \frac{\pi}{4}\right)$

Multiple Representations

The coordinates $(r, \theta) and\ (-r, \theta + \pi)$ represent the same point.

The angles $\theta + 2n\pi$ where *n* is any integer, all have the same terminal side as θ, hence, each point in the plane has infinitely many representations in polar coordinates.

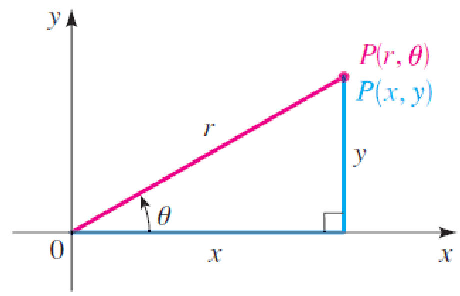


EXAMPLE – Graph the polar coordinates and two other representations for $r > 0\ and\ r < 0$

Plot the point P with polar coordinates $\left(3, \frac{\pi}{6}\right)$, and find other polar coordinates $(r,\ \theta)$for this same point: a) $r > 0, \quad 2\pi \le \theta < 4\pi$ b) $r < 0, \quad 0 \le \theta < 2\pi$ c) $r > 0, \ -2\pi \le \theta < 0$

Polar and Rectangular Coordinates

Polar and Rectangular coordinates are related as seen in adjacent the figure. We will encounter situations where we will need to relate the two systems. Using the definitions of trig functions we get the formulas below:



To change from polar to rectangular coordinates: $$\boldsymbol{x = r\cos\theta} \quad \textbf{\textit{and}} \quad \boldsymbol{y = r\sin\theta}$$
To change from rectangular to polar coordinates: $$\boldsymbol{r^2 = x^2 + y^2} \quad \textbf{\textit{and}} \quad \boldsymbol{\tan\theta = \frac{y}{x}} \quad \textbf{\textit{where}}\ \boldsymbol{x \neq 0}$$

Convert to rectangular coordinates (determine which quadrant this point is located): $\left(6, \frac{\pi}{6}\right)$	$\left(-4, -\frac{\pi}{4}\right)$
Convert the rectangular coordinates to polar coordinates (quadrant???): $\left(-1, -\sqrt{3}\right)$	

Converting equations to polar or rectangular coordinates:

Convert to polar coordinates: $4xy = 9$	Convert to rectangular coordinates: $r = 6\cos\theta$