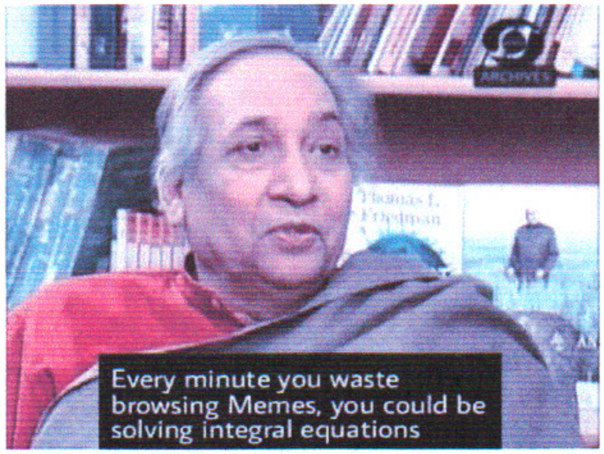
Calculus
Lesson 4.1: Antiderivatives and Indefinite Integration
Mrs. Snow, Instructor



Find a function F whose derivative is $f(x) = 3x^2$

$F(x) = x^3+5$, $F(x) = x^3-10$ $\therefore$ $F(x) = x^3+C$

From the knowledge you have of derivatives, you probably knew to work backwards by deciding what steps were taken to come up with the derivative. In other words, you would start to take the inverse of the derivative to find F. The process used to undo a derivative is called the **antiderivative.**

DEFINITION OF ANTIDERIVATIVE

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

THEOREM 4.1 REPRESENTATION OF ANTIDERIVATIVES

If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form $G(x) = F(x) + C$, for all x in I where C is a constant.

Notation for Antiderivatives (Integrals)

$y' = \frac{dy}{dx} = f(x)$

$dy = f(x)dx$

to find y

$$y = \int f(x)dx = F(x) + C$$

Variable of Integration

Integrand

Constant of Integration

So: $\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$

Basic Integration Rules

Differentiation Formula	*Integration Formula*	
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$	
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$	
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$	
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$ $\longleftrightarrow$	$*$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$	
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$	
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$	

Integrating

$\int 3x\,dx$

Basic Power rule

$= 3\int x\,dx$

$= 3\frac{x^2}{2} + C$

$= \frac{3}{2}x^2 + C$

$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx$ $+1$

$= \frac{x^{-2}}{-2} + C$

$= -\frac{1}{2x^2} + C$

$\int \sqrt{x}\,dx = \int x^{1/2}\,dx$

$= \frac{x^{3/2}}{3/2} + C$

$= \frac{2}{3}x^{3/2} + C$

$\int 2\sin x\,dx = 2\int \sin x\,dx$

$= -2\cos x + C$

More Integrating

$\int dx$

$= x + C$

$\int (x+2)\,dx = \int x\,dx + \int 2\,dx$

$= \frac{1}{2}x^2 + 2x + C$

$\int (3x^4 - 5x^2 + x\,dx$

$\frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + C$

rewrite!

$\int \frac{\sin x}{cox^2 x}\,dx = \int \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)dx$

$= \int \sec x \tan x\,dx$

$= \sec x + C$

Finding a Particular Solution

$F'(x) = \frac{1}{x^2}$ Find $F(x)$ if $F(1) = 0$

$\int \frac{1}{x^2}dx = \int x^{-2}dx$

$= -x^{-1} + C$

$F(x) = -1/x + C$

$F(1) = 0 \Rightarrow 0 = \frac{-1}{1} + C$

$1 = C$

$\therefore F(x) = \frac{-1}{x} + 1$

Solving a Vertical Motion Problem

- A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet.
  - Find the position function giving the height s as a function of the time t.
  - When does the ball hit the ground?

Initial conditions: $t = 0$

$S(0) = 80$ ft

$S'(0) = V(0) = 64 \frac{ft}{sec}$

* English units acceleration $= -32\ ft/sec^2$

So: $S''(t) = -32$

$S'(t) = \int -32\,dt$

$S'(t) = -32t + C$

$S'(0) = 64 = -32(0) + C$

$C = 64$

$S'(t) = -32t + 64$

$s(t) = \int (-32t + 64)\,dt$

$s(t) = -16t^2 + 64t + C$

$S(0) = 80 = -16(0) + 64(0) + C$

$C = 80$

$\rightarrow$ $\boxed{S(t) = -16t^2 + 64 + 80}$

$S(t) = 0 = -16(t^2 - 4t - 5)$

$0 = -16(t+1)(t-5)$

$\boxed{\text{ball hits ground at } t = 5 \text{ sec}}$