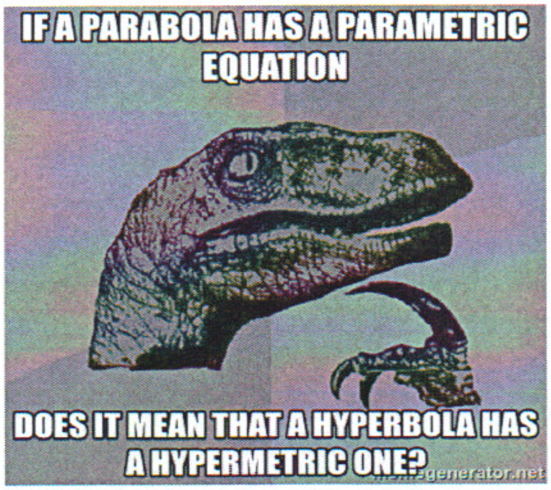
pPrecalculus

Lesson 10.7: Plane Curves and Parametric Equations

Mrs. Snow, Instructor



Think of a point moving in a plane through time. The x- and y- coordinates of the point will then be a function of time. So:

Let $x = f(t)$ *and* $y = g(t)$ where f *and* g are two functions whose common domain is some interval, I . The collection of points defined by

$$(x, y) = (f(t), g(t))$$

is called a **plane curve.** The equations

$$x = f(t) \qquad y = g(t)$$

where t is in I are **parametric equations** for the curve. the variable t is called **parameter.**

Graphing a Curve Defined by Parametric Equations: Notice that for every value of t, we get a point on the curve.

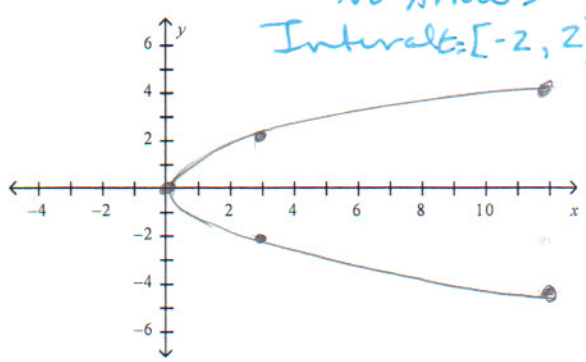
$x = 3t^2 \qquad y = 2t$

$-2 \le t \le 2$

t	x	y
-2	12	-4
-1	3	-2
0	0	0
1	3	2
2	12	4

plot x-y ordered pairs

No Arrows - Interval: [-2, 2]



Now find the rectangular equation for the parametric curve.

$x^2 = 3t^2 \qquad y = 2t$

$t = \frac{y}{2}$

$x^2 = 3\left(\frac{y}{2}\right)^2$

$x^2 = \frac{3y^2}{4}$

$\boxed{y^2 = \frac{4}{3}x}$

+ Solve "simpler" equation for t and substitute in other equation
+ Simplify solving for y.

Eliminating the Parameter: (Just did this)

Often a curve given by parametric equations can also be represented by a single rectangular equation in x and y. The process of finding this equation is called eliminating the parameter.

Find the rectangular equation for the plane curve defined by the parametric equations. Determine the domain of x.

$x = 4t,\ y = t - 3 \quad -2 \le t \le 2$

$t = \frac{x}{4}$ $\boxed{y = \frac{x}{4} - 3}$

Domain:
x-values are determined by the t-values:
find x at $t = -2$ & $t = 2$

$x = 4(-2) = -8$

$x = 4(2) = 8$

Domain: $[-8, 8]$

Solve y equation for t to eliminate the parameter

$y + 3 = t$

$x = 4(y+3)$

$x = 4y + 12$

$x - 12 = 4y$

$\boxed{y = \frac{x}{4} - 3}$

Find the rectangular equation of the curve whose parametric equations are:

$x = 4\cos t,\ and\ y = 3\sin t \qquad -0 \le t \le 2\pi$

yikes! !!

① Solve for sine & cosine:

$\cos t = \frac{x}{4} \Rightarrow \cos^2 t = \frac{x^2}{16}$

$\sin t = \frac{y}{3} \Rightarrow \sin^2 t = \frac{y^2}{9}$

② we have a trig identity involving sine & cosine.

$\sin^2 t + \cos^2 t = 1$

③ Now substitute

$\frac{y^2}{9} + \frac{x^2}{16} = 1$ ← biggest

④ What do we have??

Ellipse - horizontal $a = 4$
$b = 3$