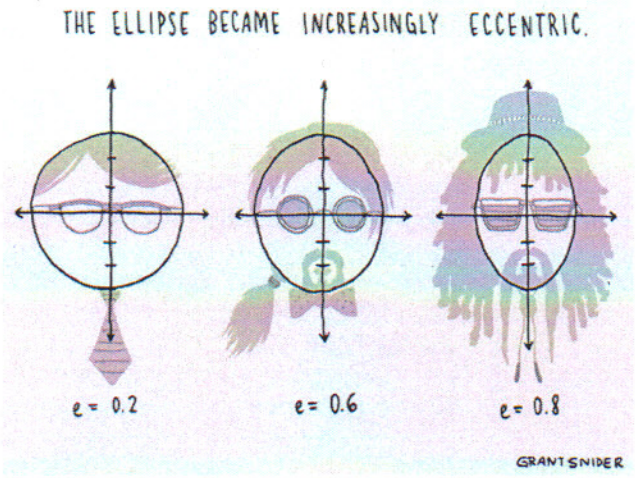
Precalculus

Lesson 10.3: The Ellipse

Mrs. Snow, Instructor



Ellipse: the collection, or locus, of all points in the plane, the sum of whose distances from two fixed points, called the foci, is constant.

Major axis it's the x-axis:

Equation of an Ellipse: Center at (0, 0); Major Axis along the *x*-Axis

An equation of the ellipse with center at $(0, 0)$, foci at $(-c, 0)$ and $(c, 0)$, and vertices at $(-a, 0)$ and $(a, 0)$ is

biggest number

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2 \qquad (2)$$

Relationship of axes & foci

Biggest denominator is major axis

a = center to vertex

Major axis is the y-axis:

Equation of an Ellipse; Center at (0, 0); Major Axis along the *y*-Axis

An equation of the ellipse with center at $(0, 0)$, foci at $(0, -c)$ and $(0, c)$, and vertices at $(0, -a)$ and $(0, a)$ is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2 \qquad (3)$$

Biggest denominator with y.

Finding an equation of an Ellipse

Find an equation of an ellipse with center at the origin, one focus at (3, 0), and a vertex at (−4, 0). Graph.

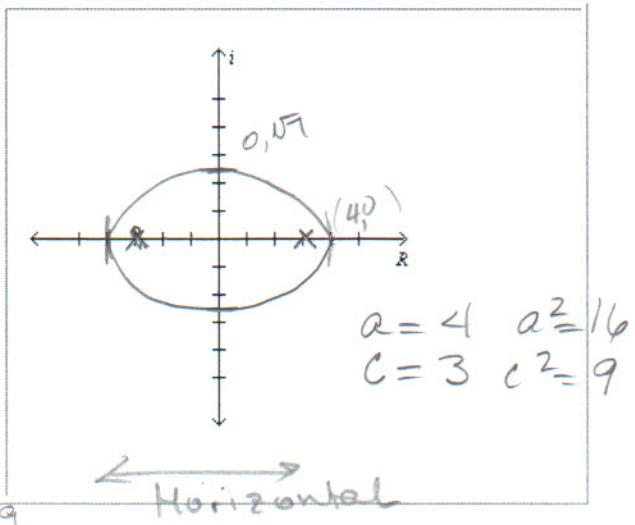
Build the equation:

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

$b^2 = a^2 - c^2$

$b^2 = 16 - 9 = 7 \quad b = \pm\sqrt{7}$

$4 < \sqrt{7} < 9$



$a = 4 \quad a^2 = 16$

$c = 3 \quad c^2 = 9$

Analyze the Equation of an Ellipse

Analyze (find the center foci and vertices)

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a^2 = 25$

$a = 5$

major axis ⇒ x

Biggest

small

vertical

$b^2 = 9$

$b = 3$

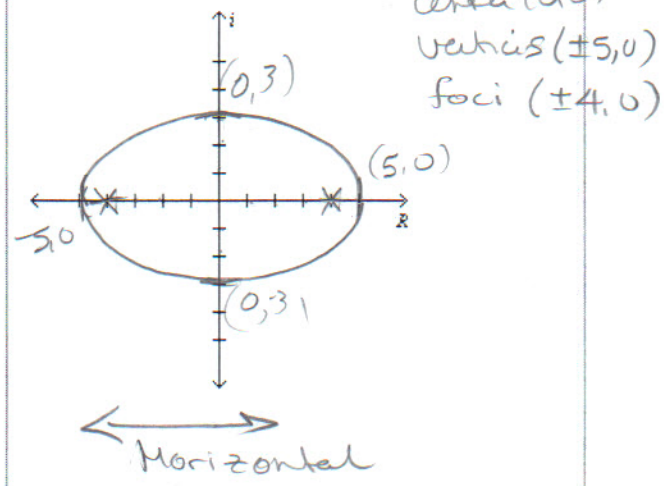
$b^2 = a^2 - c^2$

$9 = 25 - c^2$

$-16 = -c^2$

$16 = c^2$

$4 = c$



Center (0,0)

Vertices (±5,0)

foci (±4,0)

Format!

$\frac{9x^2}{9} + \frac{y^2}{9} = \frac{9}{9}$

$\frac{x^2}{1} + \frac{y^2}{9} = 1$

↑ vertical

$a^2 = 9$

$a = 3$

$b^2 = 1$

$b = 1$

$b^2 = a^2 - c^2$

$1 = 9 - c^2$

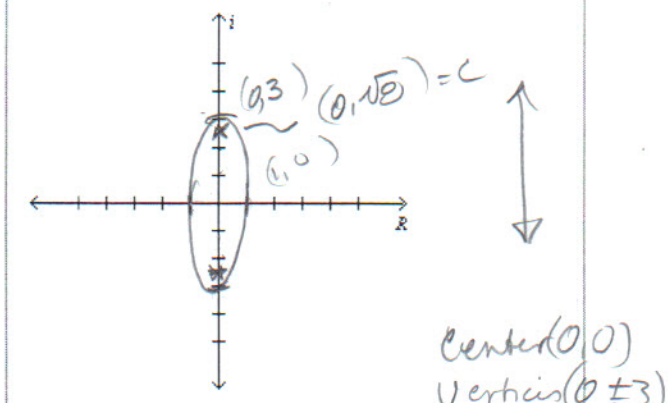
$-8 = -c^2$

$c^2 = 8$

$c = \pm 2\sqrt{2}$

$\pm\sqrt{8}$

$2 < c < 3$



Center (0,0)

Vertices (0, ±3)

Find an equation of the ellipse having one focus at $(0, 2)$ and vertices at $(0, -3)$ *and* $(0, 3)$. Graph.

$c = 2 \quad c^2 = 4$

$a = 3$

$a^2 = 9$

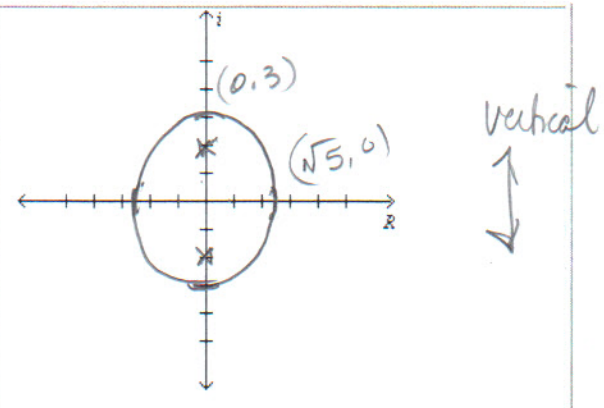
$b^2 = a^2 - c^2$

$b^2 = 9 - 4 = 5$

$b = \pm\sqrt{5}$

$2 < b < 3$

$$\frac{x^2}{5} + \frac{y^2}{9} = 1$$



vertical

Ellipses with centers at (h, k)

Equations of an Ellipse: Center at (h, k); Major Axis Parallel to a Coordinate Axis

Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to the x-axis	$(h + c, k)$	$(h + a, k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$
		$(h - c, k)$	$(h - a, k)$	$a > b > 0$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to the y-axis	$(h, k + c)$	$(h, k + a)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,$
		$(h, k - c)$	$(h, k - a)$	$a > b > 0$ and $b^2 = a^2 - c^2$

Find an equation of an ellipse with center at $(2, -3)$, (h, k) one focus at $(3, -3)$, and one vertex at $(5, -3)$. Graph

Horizontal alignment

$$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{8} = 1$$

$a^2 \qquad b^2$

$b^2 = a^2 - c^2$

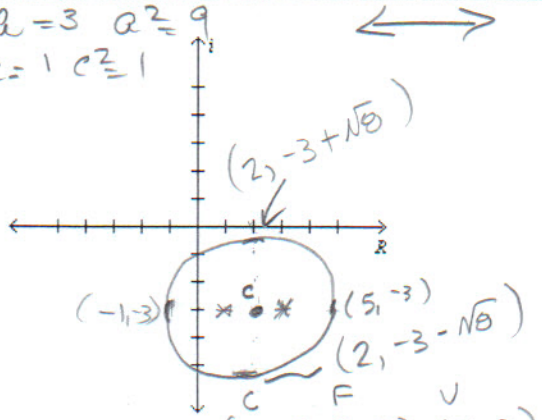
$b^2 = 9 - 1 = 8 = b^2$

$b = \pm\sqrt{8}$

$2 < b < 3$

$a = 3 \quad a^2 = 9$

$c = 1 \quad c^2 = 1$



$c = 1 \quad a = 3$

$c^2 = 1 \quad a^2 = 9$

Analyze the equation:

$$4x^2 + y^2 - 8x + 4y + 4 = 0$$

$+4$ $+4$

$4x^2-8x+4 + y^2+4y+4 = -4$

$4(x^2-2x+1^2) + (y^2+4y+2^2) = -4$

$+4+4$

$\frac{1}{2}(-2)$ $\frac{1}{2}(4)$

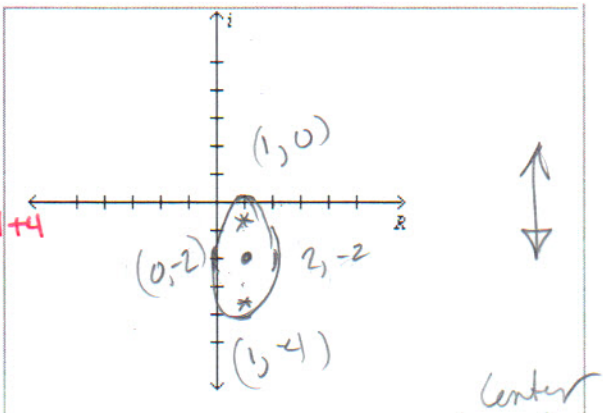
$\frac{4(x-1)^2}{4} + \frac{(y+2)^2}{4} = \frac{4}{4}$

$\frac{(x-1)^2}{1} + \frac{(y+2)^2}{4} = 1$

Center $(1,-2)$

Vertices $(1,0)$ $(1,-4)$

foci $(1, -2 \pm \sqrt{3})$



center

$(h,k) = (1,-2)$

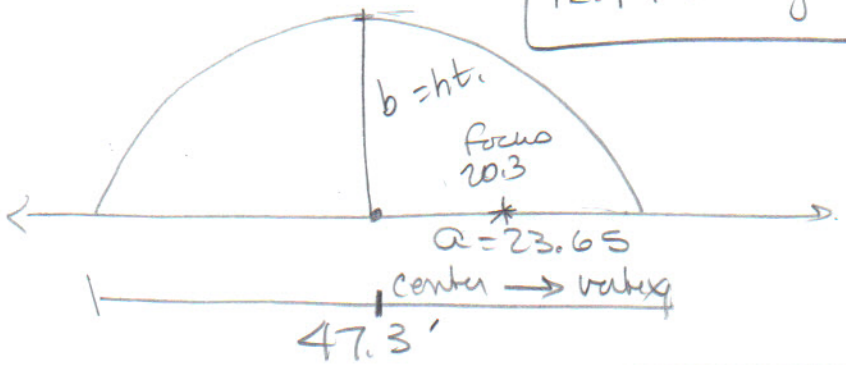
$a^2 = 4$

$b^2 = 1$

$1 = 4 - c^2$ $c^2 = 3$ $c = \pm\sqrt{3}$

The whispering gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center?

12.1 ft height



$b^2 = 23.65^2 - 20.3^2$

$b^2 = 147.2325$

$b = \sim 12.1$ ft

$$\frac{x^2}{23.65^2} + \frac{y^2}{12.12^2} = 1$$