CHAOS - Review for Related Rates & Optimization Test

1. The radius *r* and *A* of a circle are related by the equation $A = \pi r^2$. Write an equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$.

2. The radius *r* and surface area *S* of a sphere are related by the equation $S = 4\pi r^2$. Write an equation that relates $\frac{dS}{dt}$ to $\frac{dr}{dt}$.

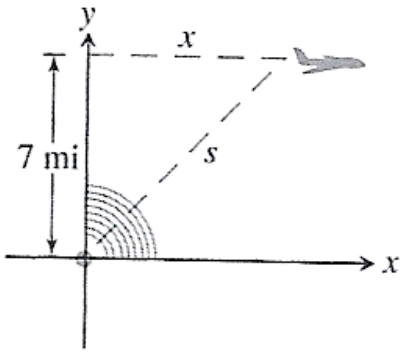
3. When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/sec. At what rate is the plate's area increasing when the radius is 50cm?

4. The length ℓ of a rectangle is decreasing at the rate of 2cm/sec while the width *w* is increasing at the rate of 2cm/sec. When $l = 12cm$ and w=5cm, find the rates of change of

a) the area b) the perimeter, and c) the length of a diagonal of the rectangle.

5. A spherical balloon is inflated with helium at the rate of $100\pi\, ft^3 / \min$.
   a) How fast is the balloon's radius increasing at the instant the radius is 5ft?
   b). How fast is the surface area increasing at that instant?

6. An airplane is flying at an altitude of 7 miles and passes directly over a radar antenna as shown in the figure. When the plane is 10 miles from the antenna (*s=10*), the radar detects that the distance *s* is changing at the rate of 300 mph. What is the speed of the airplane at that moment?



7. Mack flies a kite at a height of 300 feet, the wind carrying the kite horizontally away at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 feet away from her?

8. The volume of a right circular cone of radius *r* and height *r* is given by $V = \frac{\pi}{3} r^3$. How fast is the volume changing with respect to changes in *r* when the radius is equal to 2 feet?

9. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

10. The management of a large store wishes to add a fenced-in rectangular storage yard of 20,000 square feet, using the building as one side of the yard. Find the minimum amount of fencing that must be used to enclose the remaining 3 sides of the yard.

11. A farmer has 160 feet of fencing to enclose 2 adjacent rectangular pig pens. What dimensions should be used so that the enclosed area will be a maximum?

12. An open box is to be made from a rectangular piece of cardboard, 7 inches by 3 inches, by cutting equal squares from each corner and turning up the sides.
   a) Write the volume, V, as a function of the edge of the square, x, cut from each corner.
   b) Then use the graph of the function to estimate the size of the square that should be cut from each corner and the volume of the largest such box.

13. The sum of two nonnegative numbers is 20. Find the numbers if
   a) the sum of their squares is as large as possible; as small as possible.
   b) one number plus the square root of the other is as large as possible; as small as possible.

14. What is the largest possible area for a right triangle whose hypotenuse is 5 cm long, and what are the dimensions?

15. What is smallest perimeter possible for a rectangle whose area is 16 square inches, and what are its dimensions?

16. A rectangle has its base on the x-axis, and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

17. You are designing a rectangular poster to contain 50 square inches of printing with a 4-inch margin at the top and bottom and a 2-inch margin at each side. What overall dimensions will minimize the amount of paper used?

18. A tank is in the form of an inverted cone 8 meters across the top and 6 meters deep. Water is flowing into the tank at the rate of $10m^3 / h$. At what rate is the depth of the water in the tank changing when it is 5 meters deep?

19. The motion of a particle is given by $s = t^3 - 6t^2 + 9t \quad (t \geq 0)$, where *s* is measured in meters and *t* in seconds. A) Find the velocity and acceleration of the particle at time *t.* B) Find when the particle is moving to the left and to the right. C) Find the positions of the particle when the particle is instantaneously at rest. D) Indicate the motion of the particle in a diagram.