Calculus
Lesson 3.5: Limits at Infinity
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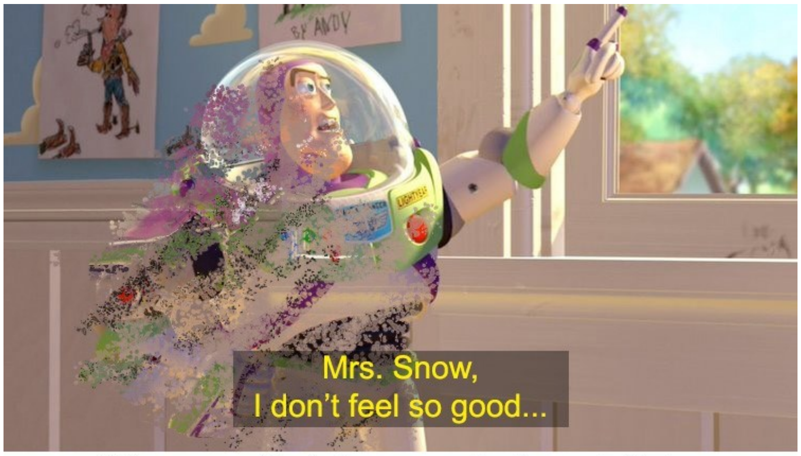

When you find there are limits at infinity

In this section we will look at the "end behavior" of a function on an infinite interval.

DEFINITION OF A HORIZONTAL ASYMPTOTE

The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x\to-\infty} f(x) = L \quad \text{or} \quad \lim_{x\to\infty} f(x) = L.$$

THEOREM 3.10 LIMITS AT INFINITY

If r is a positive rational number and c is any real number, then

$$\lim_{x\to\infty} \frac{c}{x^r} = 0.$$

Furthermore, if x^r is defined when $x < 0$, then

$$\lim_{x\to-\infty} \frac{c}{x^r} = 0.$$

Finding a Limit at infinity

$$\lim_{x\to\infty}\left(5 - \frac{2}{x^2}\right).$$

$$\lim_{x\to\infty} \frac{2x - 1}{x + 1}.$$

GUIDELINES FOR FINDING LIMITS AT $\pm\infty$ OF RATIONAL FUNCTIONS

1. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is *equal to* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function does not exist.

In regard to #3, when you encounter an indeterminate form you should divide the numerator and denominator by the **highest power of x in the denominator**.

$$\lim_{x\to\infty}\frac{2x+5}{3x^2+1} \qquad \lim_{x\to\infty}\frac{2x^2+5}{3x^2+1} \qquad \lim_{x\to\infty}\frac{2x^3+5}{3x^2+1}$$

A Function with Two Horizontal Asymptotes

a. $\lim_{x\to\infty}\frac{3x-2}{\sqrt{2x^2+1}}$

b. $\lim_{x\to-\infty}\frac{3x-2}{\sqrt{2x^2+1}}$

Limits Involving Trig Functions $\lim_{x \to \infty} \sin x$	$\lim_{x \to \infty} \frac{\sin x}{x}$

Finding Infinite Limits at Infinity
Many functions do not approach a finite limit as x increases or decreases without bound. As an example, polynomial functions do not have a finite limit at infinity.

$\lim_{x \to \infty} x^3$	$\lim_{x \to -\infty} x^3$

When evaluating a rational function, use long division to rewrite the improper rational function as the sum of a polynomial and a rational function.

$\lim_{x \to \infty} \frac{2x^2 - 4x}{x+1}$	$\lim_{x \to -\infty} \frac{2x^2 - 4x}{x+1}$