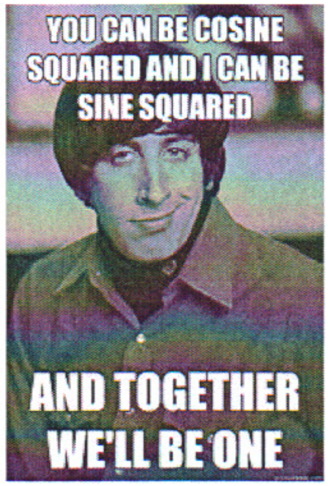
Precalculus

Lesson 7.5: Sum and Difference Formulas

Mrs. Snow, Instructor



We continue with our study of more trigonometric identities:

Sum and Difference Formlas

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

C for Cosine
C for Change sign +/−
Changing

Same

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

S for Sine
S for Same sign +/−

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

The sum and difference identities may be used to find the exact value of angles not found on the unit circle.

Think about Unit Circle Angles:

$\cos 75°$

$\cos 75 = \cos(30+45)$

$= \cos 30 \cos 45 - \sin 30 \sin 45$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

MathXL: $\frac{1}{4}(\sqrt{6}-\sqrt{2})$

$\cos\frac{\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$, $\frac{\pi}{4} = \frac{3\pi}{12}$ $\frac{\pi}{6} = \frac{2\pi}{12}$

$\cos\frac{\pi}{12} = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) =$

$\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} =$

$\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} =$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

MathXL: $\frac{1}{4}(\sqrt{2}+\sqrt{6})$

$\sin \frac{7\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$, $\frac{\pi}{4} = \frac{3\pi}{12}$ $\frac{\pi}{6} = \frac{2\pi}{12}$

$\sin \frac{7\pi}{12} = \sin\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) =$

$\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4} =$

$\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\frac{\sqrt{2}}{2} =$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

Mathxl $\frac{1}{4}(\sqrt{6}+\sqrt{2})$

$\sin 80° \cos 20° - \cos 80° \sin 20°$

$\sin(80-20) = \sin 60$

$= \frac{\sqrt{3}}{2}$

Given: $\sin\alpha = \frac{4}{5}$, $\pi < \alpha < \frac{\pi}{2}$ ← Quad II and

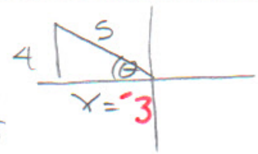
$\sin\beta = -\frac{2\sqrt{5}}{5}$, $\pi < \beta < \frac{3\pi}{2}$ ← Quad 3

Use triangles & So-Ca-TOA

find $\cos\alpha$

Know:

$\sin a = \frac{4}{5} = \frac{O}{H}$



$\cos a = \frac{-3}{5}$

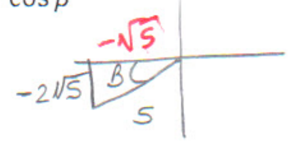
$\frac{A}{H}$

$\cos\beta$

Know

$\sin B = \frac{O}{H}$

$\frac{-2\sqrt{5}}{5}$



$x^2 + 20 = 25$

$x^2 = 5$

$x = \sqrt{5}$

$\cos B = -\frac{\sqrt{5}}{5}$

$\frac{A}{H}$

$\cos(\alpha+\beta)$ $\cos a \cos B - \sin a \sin B$

$\left(\frac{-3}{5}\right)\left(\frac{-\sqrt{5}}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{-2\sqrt{5}}{5}\right)$

$\frac{3\sqrt{5} + 8\sqrt{5}}{5} = \frac{11\sqrt{5}}{5}$

$\sin(\alpha + \beta)$ $= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\left(\frac{4}{5}\right)\left(-\frac{\sqrt{5}}{5}\right) + \left(\frac{-3}{5}\right)\left(\frac{-2\sqrt{5}}{5}\right)$

$\frac{-4\sqrt{5} + 6\sqrt{5}}{25} = \frac{2\sqrt{5}}{25}$

Establish the identities (prove):

$$\frac{\cos(\alpha - \beta)}{\sin\alpha \sin\beta} = \cot\alpha \cot\beta + 1$$

LHS

$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta} =$

Numerator + numerator
over
common Denominator

$\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$

$\cot\alpha\cot\beta + 1 = \text{RHS}$

QED

$$\tan(\theta + \pi) = \tan\theta$$

LHS

$\tan(\theta + \pi) =$

$$\frac{\tan\theta + \cancelto{0}{\tan\pi}}{1 - \cancelto{0}{\tan\theta\tan\pi}} =$$

$\tan\pi = \frac{y}{x} = 0$

$\frac{\tan\theta}{1} = \text{RHS QED}$

$$\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$$

LHS:

$$\frac{\tan\theta + \cancel{\tan\frac{\pi}{2}}}{1 - \tan\theta\tan\frac{\pi}{2}}$$ yikes undefined

cool - just like last example. or is it?

$\tan\frac{\pi}{2} = u$

Rewrite:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

LHS - take 2

$$\frac{\sin(\theta + \frac{\pi}{2})}{\cos(\theta + \frac{\pi}{2})} =$$

$$\frac{\cancelto{0}{\sin\theta\cos\frac{\pi}{2}} + \cos\theta\cancelto{1}{\sin\frac{\pi}{2}}}{\cancelto{0}{\cos\theta\cos\frac{\pi}{2}} - \sin\theta\cancelto{1}{\sin\frac{\pi}{2}}} =$$

$$-\frac{\cos\theta}{\sin\theta} =$$

$-\cot\theta = \text{RHS}$

Find the exact value of:

Flash Back!

$$\sin\left(\underbrace{\cos^{-1}\frac{1}{2}}_{\theta} + \underbrace{\sin^{-1}\frac{3}{5}}_{B}\right) = \text{so: } \sin(\theta + B)$$

$\cos^{-1}\frac{1}{2} = \theta$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\sin^{-1}\frac{3}{5} = B$

$\sin B = \frac{3}{5} = \frac{O}{H}$

$\cos B = \frac{A}{H} = \frac{4}{5}$

$\sin(\theta + B) =$

$\sin\theta\cos B + \cos\theta\sin B =$

$\left(\sin\frac{\pi}{3}\right)\left(\frac{4}{5}\right) + \cos\frac{\pi}{3}\left(\frac{3}{5}\right) =$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) =$

$\frac{4\sqrt{3}}{10} + \frac{3}{10} =$

$$\underline{\underline{\frac{4\sqrt{3}+3}{10}}}$$