Precalculus

Lesson 7.4: Trigonometric Identities

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In Chapter 6 we were introduced to the following trigonometric identities. These basic identities not only need to me memorized, but need to be second nature to you just as knowing your basic shapes and colors!!

(textbook pg.469)

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

Even–Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Simplify using trigonometric identities:

a) simplify by rewriting in terms of sine and cosine: $\frac{\cot\theta}{\csc\theta}$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} \div = \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \cos\theta$$

b) rewrite using a common denominator: $\frac{1+\sin u}{\sin u} + \frac{\cot u - \cos u}{\cos u}$

Distribute

$$= \frac{(1+\sin u)(\cos u)}{(\sin u)(\cos u)} + \frac{(\overset{\frac{\cos\theta}{\sin\theta}}{\cot u} - \cos u)}{(\cos u)} \frac{(\sin u)}{(\sin u)}$$

$$= \frac{\cos u + \cancel{\sin u \cos u}}{\sin u + \cos u} + \frac{\frac{\cos u}{\cancel{\sin u}}(\cancel{\sin u}) - \cancel{\cos u \sin u}}{\cos u \sin u}$$

$$= \frac{\cos u + \cos u}{\sin u \cos u} = \frac{2\cancel{\cos u}}{\sin u \cancel{\cos u}} = \frac{2}{\sin u}$$

$$\boxed{= 2\csc u}$$

Establish an Identity

To establish (or prove) an identity, *transform one side of the equation so that it is the same as the other side.* Do not perform the same operations on both sides of the equation, work with only one side. *Work with only one side of the equation. Oh!! And did I mention work only on one side of your equation?!?!?!?!*

1. Start with one side: generally pick the more complicated side and transform it into the other side.
2. Work vertically downward.
3. Use algebra and the fundamental identities to simplify expressions. Bring fractional expressions to a common denominator and factor.
4. Consider converting trig functions to sines and cosines or convert the expression to the trig functions that appear on the other expression.

Establish the following identities (this means we are going to prove that the LHS is equivalent to the RHS or RHS is equivalent to the LHS:

(Even/odd Trig ID's)

$\csc\theta \cdot \tan\theta = \sec\theta$	$sin^2(-\theta) + cos^2(-\theta) = 1$
LHS	LHS
$\frac{1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\cos\theta} =$	$[\sin(-\theta)]^2 + [\cos(-\theta)]^2 =$
$\frac{1}{\cos\theta} =$	$[-\sin\theta]^2 + [\cos\theta]^2 =$
$\sec\theta = RHS$	$\sin^2\theta + \cos^2\theta =$
QED	$1 = RHS$
	QED

$\frac{sin^2(-\theta) - cos^2(-\theta)}{\sin(-\theta) - \cos(\theta)} = \cos\theta - \sin\theta$	$\frac{1+\tan u}{1+\cot u} = \tan u$
LHS	LHS
$\frac{[\sin(-\theta)]^2 - [\cos(-\theta)]^2}{-\sin\theta - \cos\theta} =$	$\frac{1 + \frac{\sin u}{\cos u}}{1 + \frac{\cos u}{\sin u}} =$ Common Denominator
$\frac{[-\sin\theta]^2 - [\cos\theta]^2}{-\sin\theta - \cos\theta} =$	$\frac{\frac{\cos u}{\cos u} + \frac{\sin u}{\cos u}}{\frac{\sin u}{\sin u} + \frac{\cos u}{\sin u}} =$
(Difference of 2 squares)	
$\frac{\sin^2\theta - \cos^2\theta}{-\sin\theta - \cos\theta} =$	$\frac{\frac{\cos u + \sin u}{\cos u}}{\frac{\sin u + \cos u}{\sin u}} \div =$
$\frac{(\cancel{\sin\theta + \cos\theta})(\sin\theta - \cos\theta)}{-1(\cancel{\sin\theta + \cos\theta})} =$	$\frac{(\cancel{\cos u + \sin u})}{\cos u} \cdot \frac{\sin u}{(\cancel{\cos u + \sin u})} =$
$(-1)(\sin\theta - \cos\theta) =$	$\frac{\sin u}{\cos u} =$
$\cos\theta - \sin\theta = RHS$	$\tan u = RHS$
QED	QED :)

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\csc\theta$$

Common Denom:

LHS

$$\frac{\sin^2\theta + (1+\cos\theta)(1+\cos\theta)}{(\sin\theta)(1+\cos\theta)} =$$

Pythag. ID

$$\frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)} =$$

$$\frac{\overset{1}{\cancel{\sin^2\theta + \cos^2\theta}} + 1 + 2\cos\theta}{\sin\theta(1+2\cos\theta)} =$$

$$\frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)} =$$

$$\frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}} =$$

$$\frac{2}{\sin\theta} =$$

$$2\csc\theta = RHS$$

$$\frac{\tan v + \cot v}{\sec v \csc v} = 1$$

LHS

$$\frac{\frac{(\sin v)\sin v}{(\sin v)\cos v} + \frac{\cos v(\cos v)}{\sin v(\cos v)}}{\left(\frac{1}{\cos v}\right)\left(\frac{1}{\sin v}\right)} =$$

$$\frac{\frac{\sin^2 v + \cos^2 v}{\cos v \sin v}}{\frac{1}{\cos v \sin v}} =$$

$$\frac{\frac{1}{\cos v \sin v}}{\frac{1}{\cos v \sin v}} =$$

$$1 = RHS$$

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

LHS:

$$\frac{1-\sin\theta}{\cos\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} =$$ Multiply by "1"

$$\frac{1-\sin^2\theta}{(\cos\theta)(1+\sin\theta)} =$$ Pythagorean Identity

$$\frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}(1+\sin\theta)} =$$

$$\frac{\cos\theta}{1+\sin\theta} = \text{RHS}$$

QED.

OR

RHS

$$= \frac{\cos\theta}{1+\sin\theta} \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$\text{LHS} = \frac{1-\sin\theta}{\cos\theta}$$

QED