

Calculus  
 Lesson 2.5: Implicit Differentiation  
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**Implicit vs. Explicit:** What's the difference? Up to this point, most functions we have dealt with are expressed in **explicit form**, that is the variable  $y$  is **explicitly** written as a function of  $x$ , e.g.

$y = \frac{1}{x}$ . Some functions, however, are **implied** by a given equation. An implicit form of the previous example is:  $xy = 1$ . Note, the implicit form is where the dependent variable is not isolated on one side of the equation. So, no problem we simply solve the equation for  $y$  and then take the derivative.



Woops! The problem is that not all equations that we work with can be solved explicitly for  $y$  as a function of  $x$ . For example, how do we find  $\frac{dy}{dx}$  for the equation:  $x^2 - 2y^3 + 4y = 2$  ????

Soooooo, to understand how to find  $dy/dx$  implicitly, you must realize that the differentiation is taking place with respect to  $x$ . This means that when you differentiate terms involving  $x$  alone, you can differentiate as usual. However, when you differentiate terms involving  $y$ , you must apply the Chain Rule, because you are assuming that  $y$  is defined implicitly as a differentiable function of  $x$ .

Differentiate with respect to $x$ : $\frac{d}{dx} x^3$  Variables match	$\frac{d}{dx} [y^3]$  Variables don't match: use power rule and chain rule
$\frac{d}{dx} [x + 3y]$	$\frac{d}{dx} [xy^2]$

### GUIDELINES FOR IMPLICIT DIFFERENTIATION

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving  $dy/dx$  on the left side of the equation and move all other terms to the right side of the equation.
3. Factor  $dy/dx$  out of the left side of the equation.
4. Solve for  $dy/dx$ .

**Implicit Differentiation:** Find  $\frac{dy}{dx}$  given that  
 $y^3 + y^2 - 5y - x^2 = -4$

**Finding the slope of a graph implicitly**  
Determine the slope of the tangent line to the graph of  $x^2 + 4y^2 = 4$  at the point  $(\sqrt{2}, \frac{-1}{\sqrt{2}})$ .

Determine the slope of the graph of  $3(x^2 + y^2)^2 = 100xy$  at the point (3,1).

**Find the second derivative Implicitly**

Given  $x^2 + y^2 = 25$ , find  $\frac{d^2y}{dx^2}$

**Finding a tangent line to a graph**

Find the tangent line to the graph given by  $x^2(x^2 + y^2) = y^2$  at  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$