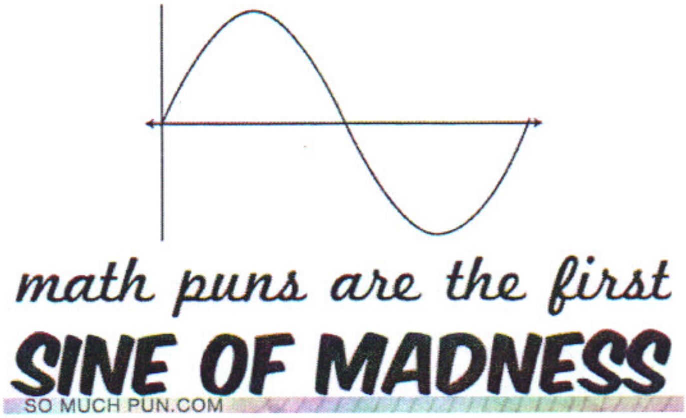
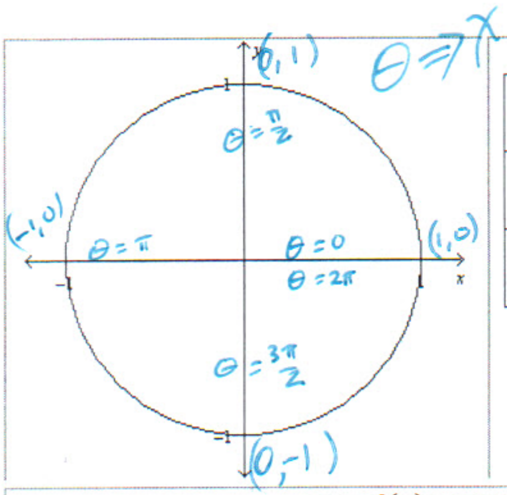
Precalculus

Lesson6.4: Graphs of the Sine and Cosine Functions

Mrs. Snow, Instructor



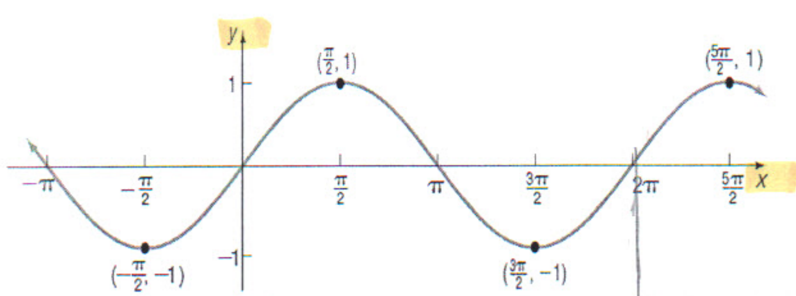
We now want to graph the trigonometric functions on the *x*-*y*-plane using *x* as our independent variable and *y*, in the form of $\sin x$, as our dependent variable. Select values for the input of x and evaluate the output of $\sin x$.



x is the Angle & in Radians

x Angle	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	$\sin 0$	$\sin\frac{\pi}{2}$	$\sin\pi$	$\sin\frac{3\pi}{2}$	$\sin 2\pi$
$y = \sin x$	0	1	0	−1	0

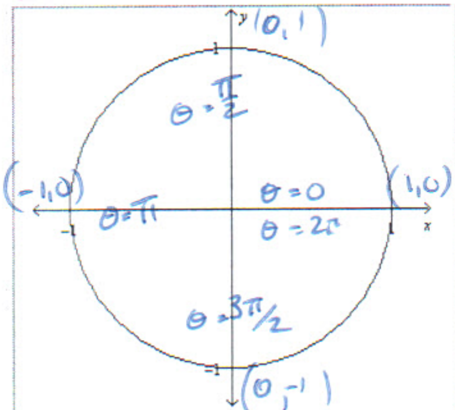
$y = f(x) = \sin x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
Amplitude: 1
x=intercepts:
$2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots\ldots$

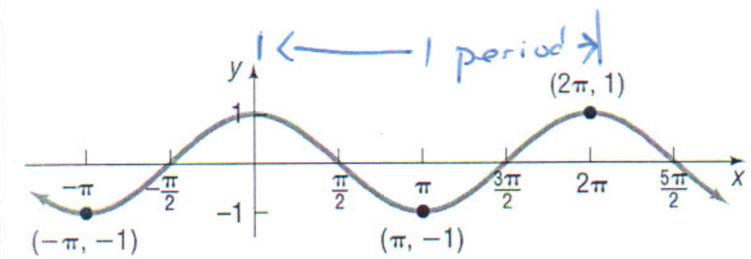
← 1 period →

The same may be done for $\cos\theta$



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	$\cos 0$	$\cos\frac{\pi}{2}$	$\cos\pi$	$\cos\frac{3\pi}{2}$	$\cos 2\pi$
$y = \cos x$	1	0	−1	0	1

$y = f(x) = \cos x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
Amplitude: 1

x-intercepts:
$-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

What we see now is a graph that is periodic. A function that repeats itself in regular intervals or periods is said to be **periodic.** The functions sine and cosine have a period of 2π, so these we see these functions repeating the output over the interval length of 2π.

Since we are shifting gears and moving away from θ *and* t, we will now use the variables x *and* y to denote the domain and range for the functions. In applications to physical situations we will need to be able to transform or shift the sine and cosine functions up/down, left/right, and narrow/wide. In actuality, we apply the principles of transformations as used in Algebra 1 and 2.

yikes!

NOT ANGULAR VELOCITY!!

Transformations in the form: $y = A\sin\omega x$ *and* $y = A\cos\omega x$

$y = A\sin\omega x + B$	$y = A\cos\omega x + B$
Amplitude: $\lvert A\rvert$ ← Vertical Stretch →	Amplitude: $\lvert A\rvert$
Period$= T = \frac{2\pi}{\omega}$ B = Vertical Slide ↑↓	Period$= T = \frac{2\pi}{\omega}$

Period is the distance for function to complete 1 cycle

Graph requirements:

1. *graph 1 period*
2. *Divide the period into 4 intervals and use the key points to graph the function*
3. *label x and y axes values*
    a. *all intercepts*
    b. *minimum and maximum*

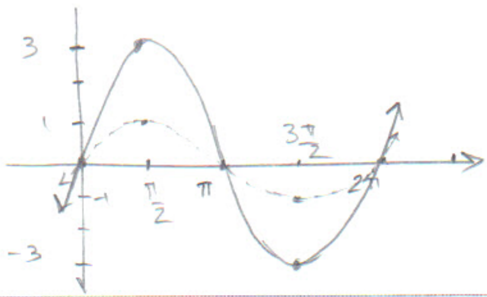
    } 5 critical points
4. *use arrows to indicate end behavior*

Graph $y = 3\sin x$, use the graph to determine the domain and range

$\omega = 1$

$Pd = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$

$A = 3$ stretch $\times 3$



Show: *5 critical pts ✓

* end behavior (arrows) ✓

* 1 pd ✓

Graphing Trigonometric Functions Using Key Points

Make a table of values for x based on the interval of the period of the function and divide it into 4 intervals. While the table is not absolutely necessary, if an error is made, the tabulation of steps will help find the error. As problems become more complicated, it too will help with graph construction. Yes, you can reason these shifts and do the plotting on the graphs directly; the table while a little more time really shows you what is happening to function especially if it gets complicated.

More complicated use table:

Graph $y = 3\sin 4x$, use the graph to determine the domain and range.

$\omega = 4$

$Pd = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

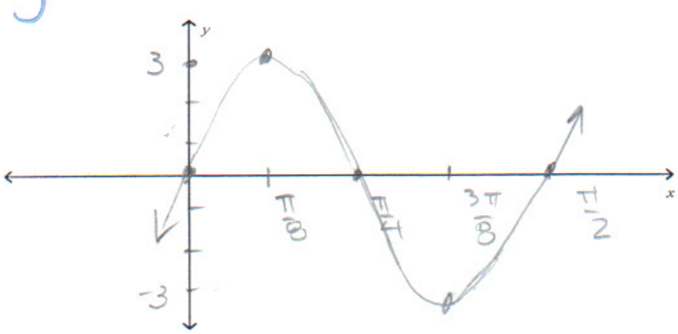
Divide period evenly from $0 \to \frac{\pi}{2}$

x	0	$\frac{\pi}{8}$ (4)	$\frac{\pi}{4}$ (4)	$\frac{3\pi}{8}$ (4)	$\frac{\pi}{2}$
$4x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin 4x$	0	1	0	-1	0
$y = 3\sin 4x$	0	3	0	-3	0

← x Graph

* Check point

← y Graph



* yes! These are Unit Circle 4 Values

Show: 1 pd ✓

5 critical pts ✓

End behavior ✓

Odd function pullout neg.

Determine the amplitude and period of and graph of $y = 2\sin\left(-\frac{\pi}{2}x\right)$

1 period of 4 divided:

graph x	0	1	2	3	4
$\frac{\pi}{2}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π *
$\sin \frac{\pi}{2}x$	0	1	0	-1	0
graph $y = -2\sin \frac{\pi}{2}x$	0	-2	0	2	0

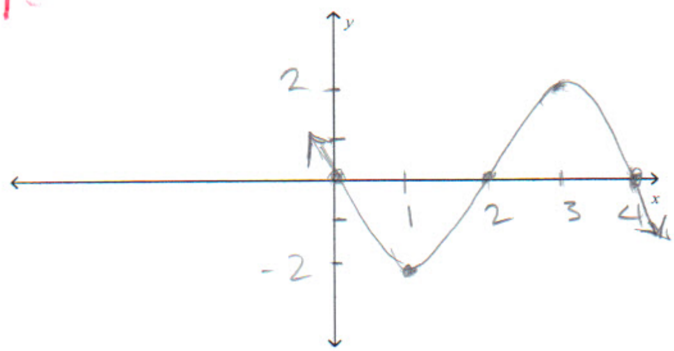
$y = -2\sin\left(\frac{\pi}{2}x\right)$

$A = -2$ ↓ flip

$\omega = \frac{\pi}{2}$

$Pd = \frac{2\pi}{\frac{\pi}{2}} = 2\cancel{\pi}\left(\frac{2}{\cancel{\pi}}\right)$

$Pd = 4$



* check! Unit Cir. axes!

Shown:
✓ End Behavior
✓ 5-crit. pts.
✓ 1 period

Graph $y = -4\cos(\pi x) - 2$ using key points.

1 period Divided!

graph x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
πx	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π *
$\cos \pi x$	1	0	-1	0	1
$-4\cos \pi x$	-4	0	4	0	-4
graph $y = -4\cos \pi x - 2$	-6	-2	2	-2	-6

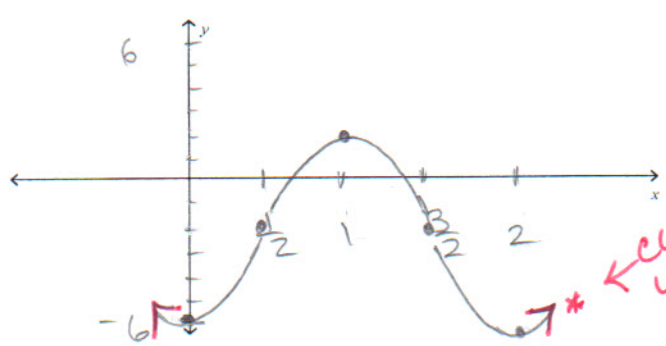
$A = -4$ ↓ flip

$\omega = \pi$

$Pd = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

-2 ↓ down

* ✓ Unit circle Values!

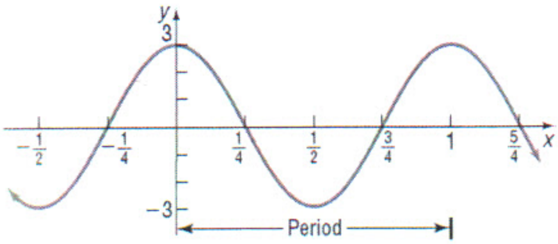


* ← curve back up

Shown:
* ✓ End behavior
✓ 5-c.p
✓ 1 pd

Remember* Shape of curve

Find an equation for the graph shown



What shape?

Sine or Cosine?

(max on y-axis)

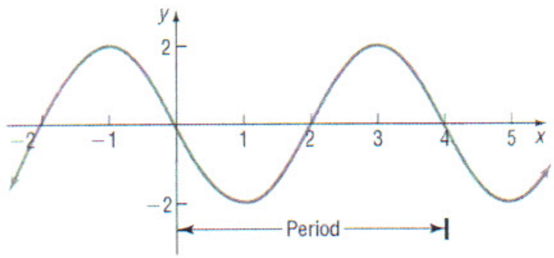
$Pd = 1 = \frac{2\pi}{\omega}$

$\omega = 2\pi$

$A = 3$

$\boxed{y = 3\cos 2\pi x}$

Find an equation for the graph shown



Sine Shape

(Cross y-axis)

$Pd = 4 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$A = -2$

$\boxed{y = -2\sin \frac{\pi}{2}x}$

Textbook pg. 405

SUMMARY **Steps for Graphing a Sinusoidal Function of the Form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ Using Key Points**

STEP 1: Determine the amplitude and period of the sinusoidal function.

STEP 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

STEP 3: Use the endpoints of these subintervals to obtain five key points on the graph.

STEP 4: Plot the five key points and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.