Calculus

Lesson: 3.3 Increasing and Decreasing Functions and the First Derivative Test

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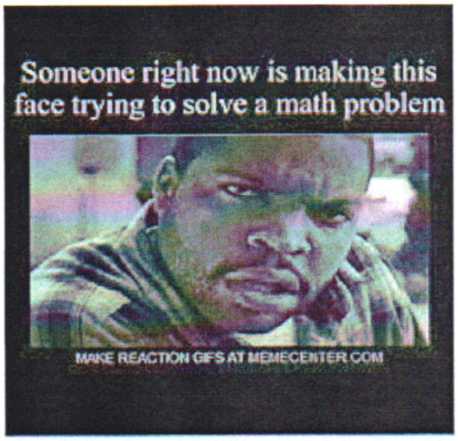
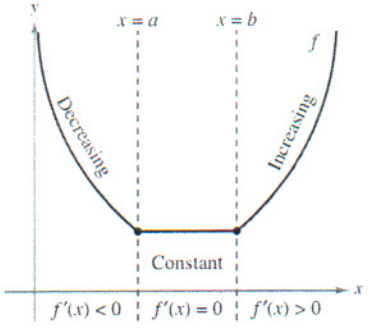


In this section we will study how derivatives can be used to classify relative extrema as either relative minima or relative maxima.

DEFINITIONS OF INCREASING AND DECREASING FUNCTIONS

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



The derivative is related to the slope of a function.

A function is increasing if, *as x moves to the right*, its graph moves up, and is decreasing if its graph moves down.

THEOREM 3.5 TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

WARNING!! If our function f is to be continuous, always remember to verify the domain of f before determining the critical points. If could be that where f′is a "DNE" critical point, f has a domain restriction!!!

Intervals of Which f is Increasing or Decreasing

Find the open intervals on which f(x) is increasing or decreasing.

$f(x) = x^3 - \frac{3}{2}x^2$

$f' = 3x^2 - 3x = 0$

$3x(x-1) = 0$

$x = 0, 1$ critical #

Are there DNE ? None

Test pt	-1	$1/2$	2
	0		1
$(3x)$	$-$	$+$	$+$
$(x-1)$	$-$	$-$	$+$
	$+$	$-$	$+$
	↗	↘	↗

Ans Incr.: $(-\infty, 0)$ & $(1, \infty)$ Decr. $(0, 1)$

GUIDELINES FOR FINDING INTERVALS ON WHICH A FUNCTION IS INCREASING OR DECREASING

Let f be continuous on the interval (a, b). To find the open intervals on which f is increasing or decreasing, use the following steps.

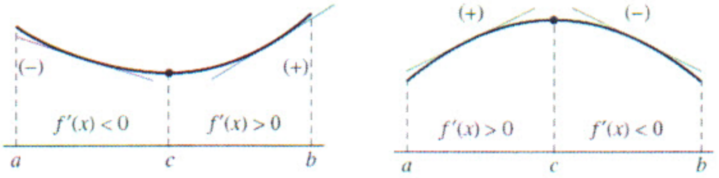
1. Locate the critical numbers of f in (a, b), and use these numbers to determine test intervals.
2. Determine the sign of $f'(x)$ at one test value in each of the intervals.
3. Use Theorem 3.5 to determine whether f is increasing or decreasing on each interval.

These guidelines are also valid if the interval (a, b) is replaced by an interval of the form $(-\infty, b)$, (a, ∞), or $(-\infty, \infty)$.

THEOREM 3.6 THE FIRST DERIVATIVE TEST

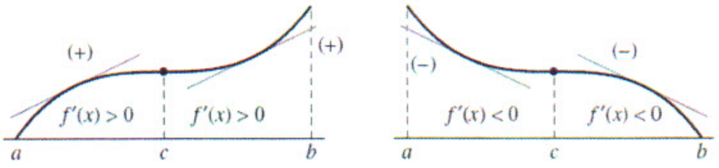
Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



Relative minimum

Relative maximum



Neither relative minimum nor relative maximum

Applying the First Derivative Test

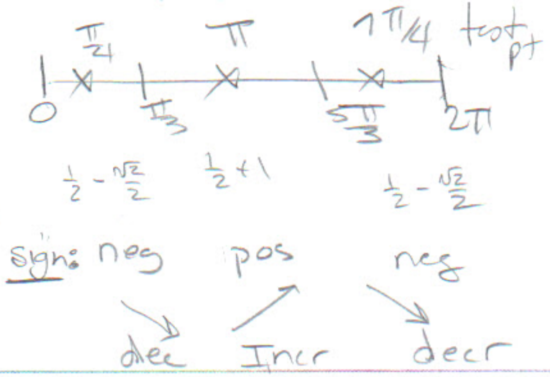
Find the relative extrema of the function f(x) in the interval (0,2π). ***(check domain of $f(x)$)***

$f(x) = \frac{1}{2}x - \sin x$

$f' = \frac{1}{2} - \cos x = 0$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ Critical #



Ans:

relative min: $x = \pi/3$

relative max: $x = \frac{5\pi}{3}$

Find the relative extrema of

$f(x) = \left(x^2 - 4\right)^{2/3}$

$f' = \frac{2}{3}(x^2-4)^{-1/3}(2x)$

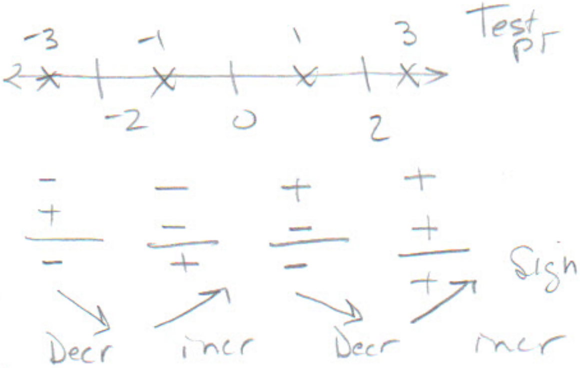
$f' = \frac{4x}{3(x^2-4)^{1/3}} = 0$

$4x = 0$ & $x^2 - 4 = 0$ (DNE)

$x = 0$ $x = \pm 2$

Critical numbers

$x = 0$ & ± 2



Ans:

rel. min: $(-2, 0)$ $(2, 0)$

rel max: $(0, \sqrt[3]{16})$

Find the relative extrema of

$f(x) = \frac{x^4+1}{x^2}$ rewrite: $= x^2 + x^{-2}$

Asymptote at $x=0$!

common denom:

$f' = 2x - \frac{2}{x^3} = \frac{2x^4 - 2}{x^3}$

$= \frac{2(x^4-1)}{x^3}$

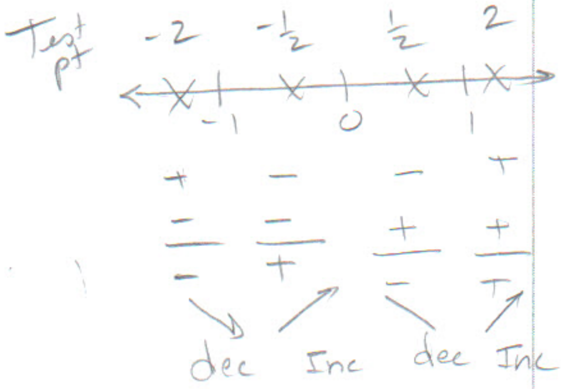
$= \frac{2(x^2+1)(x^2-1)}{x^3}$

$= \frac{2(x^2+1)(x+1)(x-1)}{x^3} = 0$ set num=0

set denom.=0

$x = 1, -1$ & $x = 0$

(DNE)



Ans:

rel. min: $(-1, 2)$

$(1, 2)$

at $x = 0$, Asymptote