## Precalculus

## Lesson6.4: Graphs of the Sine and Cosine Functions

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## math puns are the first SINE OF MADNESS

We now want to graph the trigonometric functions on the $x$ - $y$-plane using $x$ as our independent variable and $y$, in the form of $\sin x$, as our dependent variable. Select values for the input of $x$ and evaluate the output of $\sin x$.


The same may be done for $\cos \theta$

|  | $x$ <br> $\cos x$ <br> $y=\cos x$ | 0 $\cos 0$ | $\frac{\pi}{2}$ <br> $\cos \frac{\pi}{2}$ | $\pi$ $\cos \pi$ | $\frac{\frac{3 \pi}{2}}{\cos \frac{3 \pi}{2}}$ | $2 \pi$ $\cos 2 \pi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=f(x)=\cos x$ |  |  |  | Domain: $(-\infty, \infty)$ <br> Range: $[-1,1]$ <br> Period: $2 \pi$ <br> Amplitude: 1 <br> x-intercepts: $-\frac{3 \pi}{2},-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3 \pi}{2}, \frac{5 \pi}{2}, \ldots \ldots$ |  |  |

What we see now is a graph that is periodic. A function that repeats itself in regular intervals or periods is said to be periodic. The functions sine and cosine have a period of $2 \pi$, so these we see these functions repeating the output over the interval length of $2 \pi$.

Since we are shifting gears and moving away from $\theta$ and $t$, we will now use the variables $x$ and $y$ to denote the domain and range for the functions. In applications to physical situations we will need to be able to transform or shift the sine and cosine functions up/down, left/right, and narrow/wide. In actuality, we apply the principles of transformations as used in Algebra 1 and 2.

Transformations in the form: $y=A \sin \omega x$ and $y=A \cos \omega x$

| $\boldsymbol{y}=\boldsymbol{A} \sin \omega \boldsymbol{x}$ | $\boldsymbol{y}=\boldsymbol{A} \boldsymbol{\operatorname { c o s } \omega \boldsymbol { x }}$ |
| :---: | :---: |
| Amplitude: $\|A\|$ | Amplitude: $\|A\|$ |
| Period $=T=\frac{2 \pi}{\omega}$ | Period $=T=\frac{2 \pi}{\omega}$ |

## Graph requirements:

1. graph 1 period
2. Divide the period into 4 intervals and use the key points to graph the function
3. label $x$ and $y$ axes values
a. all intercepts
b. minimum and maximum
4. use arrows to indicate end behavior

Graph $\boldsymbol{y}=\mathbf{3} \sin \boldsymbol{x}$, use the graph to determine the domain and range

## Graphing Trigonometric Functions Using Key Points

Make a table of values for $x$ based on the interval of the period of the function and divide it into 4 intervals. While the table is not absolutely necessary, if an error is made, the tabulation of steps will help find the error. As problems become more complicated, it too will help with graph construction. Yes, you can reason these shifts and do the plotting on the graphs directly; the table while a little more time really shows you what is happening to function especially if it gets complicated.

Graph $y=3 \sin 4 x$, use the graph to determine the domain and range.


Determine the amplitude and period of and graph of $\boldsymbol{y}=2 \boldsymbol{\operatorname { s i n }}\left(-\frac{\pi}{2} \boldsymbol{x}\right)$

| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |



Graph $\boldsymbol{y}=-4 \cos (\boldsymbol{\pi} \boldsymbol{x})-2$ using key points.


Find an equation for the graph shown


Find an equation for the graph shown


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## SUMMARY Steps for Graphing a Sinusoidal Function of the Form $y=A \sin (\omega x)$ or

 $y=A \cos (\omega x)$ Using Key PointsStep 1: Determine the amplitude and period of the sinusoidal function.

Step 2: Divide the interval $\left[0, \frac{2 \pi}{\omega}\right]$ into four subintervals of the same length.

Step 3: Use the endpoints of these subintervals to obtain five key points on the graph.

STEP 4: Plot the five key points and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.

