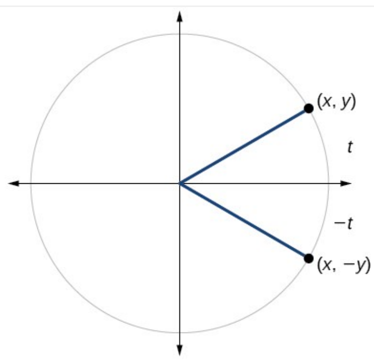
Precalculus
Lesson 6.1: Angles and Their Measure
Lesson 6.2: A Unit Circle Approach
Part 2

Lesson 6.2

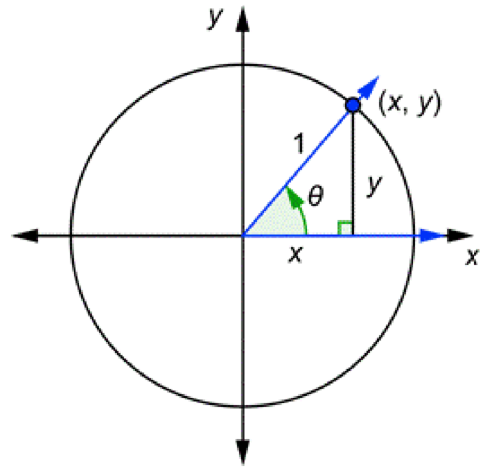
Before we look at the unit circle with respect to the trigonometric functions, we need to get some terminology down for unit circle use. Remember the **Unit Circle has a radius of 1**.

Terminal Point – For and angle in standard position, let $P = (x, y)$ be the point of the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$

Reference angle - The reference angle is always the **smallest** angle that you can make from the terminal side of an angle and the **x-axis**. The reference angle always uses the x-axis as its frame of reference. A reference angle must **be** $< 90°$ **or** $< \frac{\pi}{2}$ $\boldsymbol{rad}$**.**



Trig Functions	Reciprocal functions
$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} = y$	$\csc\ \theta = \frac{hyp}{opp} = \frac{r}{y} = \frac{1}{y}$
$\cos\ \theta = \frac{adj}{hyp} = \frac{x}{r} = x$	$\sec\ \theta = \frac{hyp}{adj} = \frac{r}{x} = \frac{1}{x}$
$\tan\ \theta = \frac{opp}{adj} = \frac{y}{x}$	$\cot\ \theta = \frac{adj}{opp} = \frac{x}{y}$



UNIT CIRCLE

The Unit Circle may be constructed using the above idea, a basic understanding of geometry, and recognizing the correlation of the arc distance (terminal point, t) and the degree measure of the angle formed with the radius

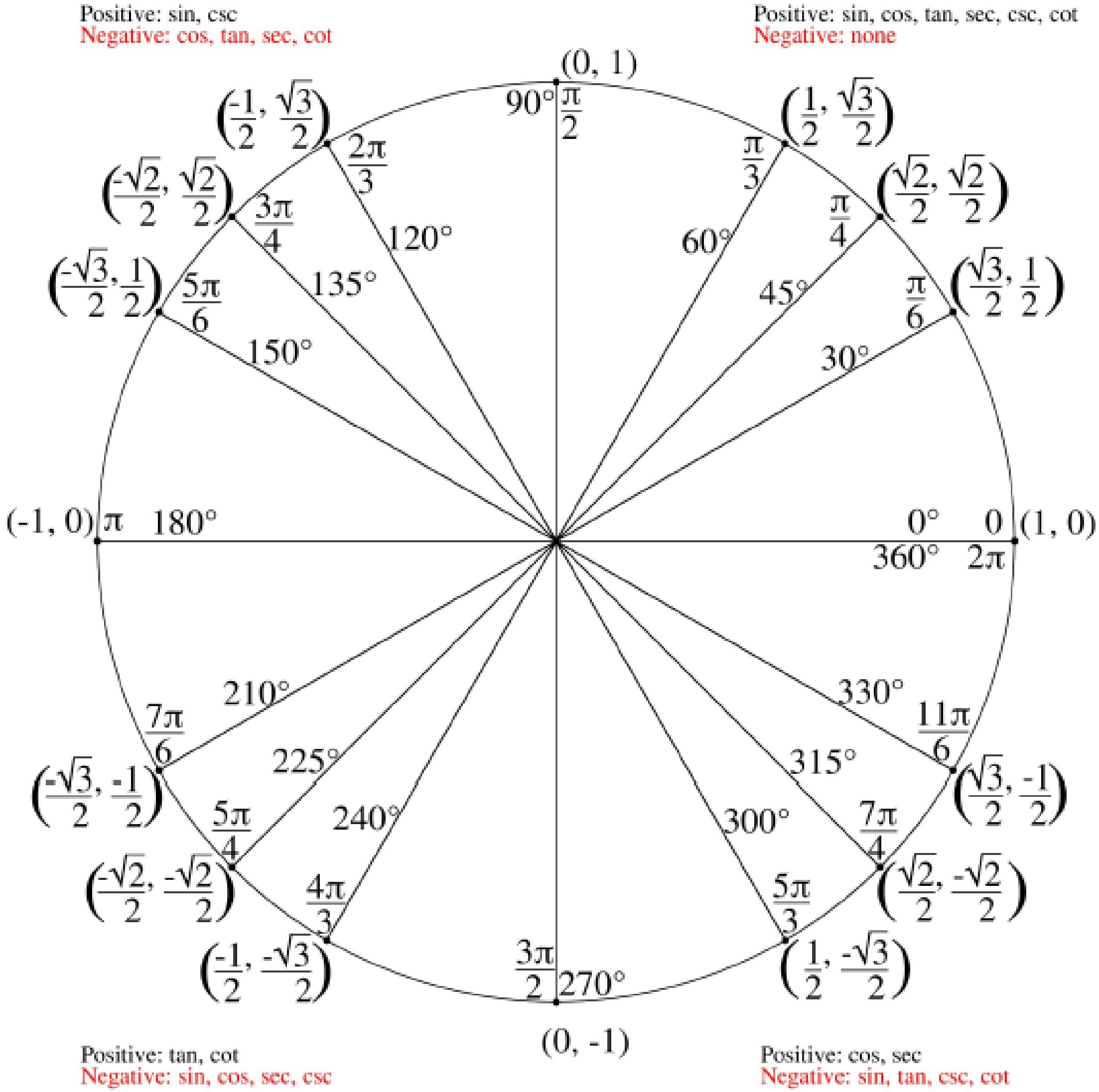
SPECIAL RIGHT TRIANGLES……

Let's review our Special Triangles: $30 - 60 - 90\ \ and\ 45 - 45 - 90$

So now we can determine the (x-y) ordered pairs at our terminal end points and complete the unit circle from last class. **(pg. 376)**

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Going back to the unit circle we built last class, we are able to fill in the missing (x, y) ordered pairs:



Let t be a real number and let $P = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ be the point of the unit circle that corresponds to t Find the values of $sin\ t,\ cos\ t,\ tan\ t,\ csc\ t,\ sec\ t, and\ cot\ t$.

Finding exact values of the Six Trigonometric Functions using a point on the Unit Circle

Find the exact values of the six trigonometric function of:

a) $\cos\frac{5\pi}{4}$

d) $\cos\frac{8\pi}{3}$

b) $\tan 315$

e) $\csc\frac{\pi}{6}$

c) $\sin(-60)$

f) $\sec 45$

Find the exact values a trigonometric function

Find the exact value of each expression

a) $sin45°cos180°$

b) $\tan\frac{\pi}{4} - \sin\frac{3\pi}{2}$

c) $(\sec\frac{\pi}{4})^2 + \csc\frac{\pi}{2}$

Using a calculator to approximate the value of a trig function:

a) $cos\ 48$

b) $csc\ 21$

c) $tan\frac{\pi}{12}$

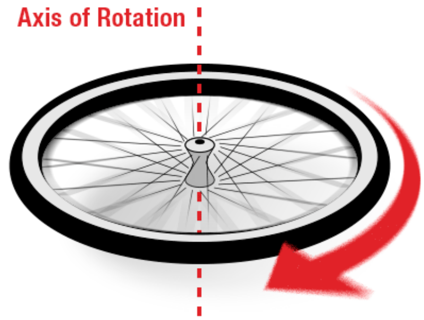
Finding the exact value of the six trig functions

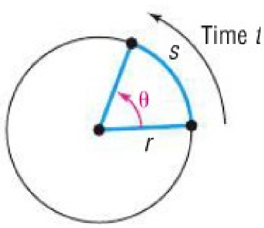
Find the exact values of each of the six trig functions of an angle $\theta\ if\ (4,-3)$ is a point on its terminal side in standard position.

Back to Lesson 6.1

CIRCULAR SPEED

When we look at an object moving in a circle, we generally want to know the speed at which it is spinning. **Angular velocity** is the rate at which the object is spinning around its axis. Angular velocity describes **the amount the angle changes** as the object rotates (or revolves) in a specific period of time.





Think about an object spinning in a circular orbit around a point. There are two ways we can describe the speed:

Linear Speed – the rate the distance travelled is changing

Angular Speed – the rate the central angle, θ is changing

Linear Speed (v): linear distance travelled over time.

$$v = \frac{s}{t}$$

- mph (miles per hour)
- Feet per second
- Inches per minute
- Kilometers per hour

Angular Speed (ω): Angle displaced over time.

$$\omega = \frac{\theta}{t}$$

- Degrees per second
- Radians per minute
- Revolutions per minute (RPM)
- Rotations per hour

If a point moves along a circle of radius r with angular speed ω, then its linear speed v is given by:

$$v = r\omega$$

A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.