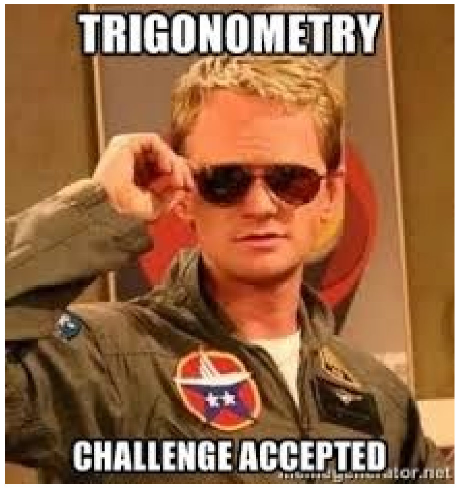
Precalculus

Lesson 6.1: Angles and Their Measure

Lesson 6.2: A Unit Circle Approach

Part 1

Mrs. Snow, Instructor

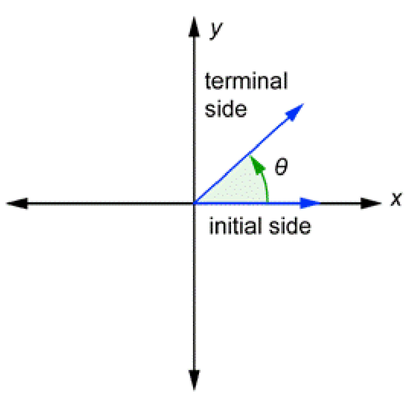


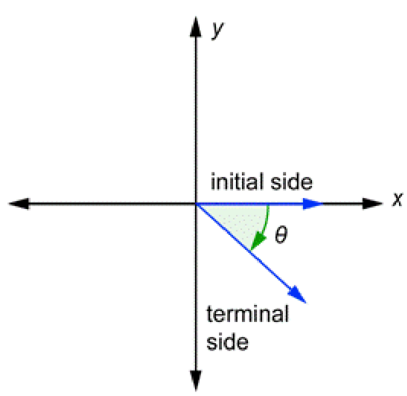
In Trigonometry we will be working with angles from
$0° \ to \ 180° \ to \ 360° \ to \ 720° \ and \ even \ more!$
We will also work with degrees that are smaller than $0°$!!

Check out Shaun White's YouTube video of his 2010 Olympic Gold Medal McTwist 1260….yep, it has 1260 degrees of total rotation. Shaun thinks beyond one circle of 360 degrees!

VOCABULARY

Standard Position – An angle is in standard position if its vertex is located at the origin with one ray on the positive x-axis. The ray on the x-axis is called the **initial side** and the other ray is called the **terminal side**.





Positive angle – A positive angle is created by rotating counterclockwise:
$360° = 1$ counterclockwise revolution

Negative angle – A negative angle is measured in the clockwise direction from the positive horizontal axis: $-360° = 1$ clockwise revolution

Draw each angle:

a) $45°$

c) $-90°$

b) $225°$

d) $405°$

Convert between degrees-minutes-seconds (DMS) and decimal measures for angles.

$$1° = 60'$$
$$1' = 60''$$

Calculator: Have the calculator in **degree mode**. For the examples below use the following key strokes. The 2nd apps allows you to select degrees and minutes. The catalog gives you the seconds

[5] [0] [2nd] [APPS] [ENTER] [6] [2nd] [APPS] [⌄] [ENTER] [2]
[1] [2nd] [0] [⌃] [⌃] [⌃] [⌃] [ENTER] [ENTER]

To convert from decimal degrees for the example given use the following key stokes:

[2] [1] [.] [2] [5] [6] [2nd] [APPS] [ENTER] [2nd] [APPS]
[⌄] [⌄] [⌄] [ENTER] [ENTER]

Convert $50°\ 6'\ 21''$ to decimal degrees. Round to four decimal places.

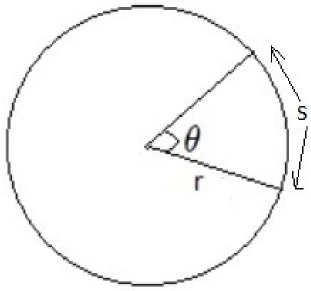
Convert $21.256°$to DMS. Round to the nearest second.

RADIANS

In geometry you studied and learned much about angle measure using degrees. Rotating in a circular path one complete revolution is said to be 360 degrees. Degrees are used to express direction off of magnetic north and angle size. While degree measurements are used in everyday activities such as construction, land surveying, or describing the exact location of a ship at sea or a jet in the sky, degrees are actually not numbers. They are a fraction of a prescribed total number of degrees to make a complete rotation. Radians are based on the measurement of the circumference of a circle. Radians are used in many science and engineering applications.

So, how many radians are in a circle?

If a circle of radius 1 is drawn with the vertex of an angle at its center, then:
The measure of this angle in radians is the length of the arc that subtends the angle.
$C = 2\pi \mathrm{r}$



Conversions

$$\theta = 360° = 2\pi\ rad \ \therefore \ 180° = \pi\ rad$$

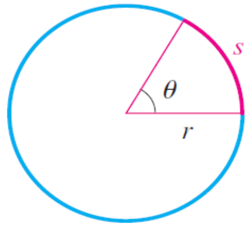
$$1° = \frac{\pi}{180} radian \ \ or, \qquad 1 = \frac{\boldsymbol{\pi\ radians}}{\boldsymbol{180\ degrees}}$$

$$1\ rad = \left(\frac{180}{\pi}\right) degrees \ \ or, \qquad \mathbf{1} = \frac{\boldsymbol{180\ degrees}}{\boldsymbol{\pi\ radians}}$$

Converting from degrees to radians

Convert each angle in degrees to radians.	Convert each agle in radians to degrees:
a) $60°$	a) $\frac{\pi}{6}\ radian$
b) $150°$	b) $\frac{3\pi}{2}\ radian$
c) $-45°$	c) $-\frac{3\pi}{4}\ radian$
d) $90°$	**d)** $3\ \ radians$

LENGTH OF A CIRCULAR ARC



What is the measure of the arc **s** in the diagram to the left? Create a proportion relating the ratio of θ to the whole circle which is $2\pi\ radians$ and the ratio of the arc length to the circumference. The proportion simplifies giving the arc length as:

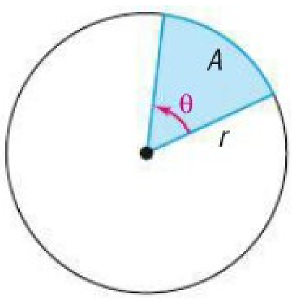
$$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$$

$$\boldsymbol{s = \theta r \quad or \quad \theta = \frac{s}{r}}$$

$\boldsymbol{\theta\ must\ be\ in\ radians}$!!!!!

Find the length of the arc of a circle of radius 2 meters subtended by a central angle of 0.25 radian.

AREA OF A CIRCULAR SECTOR



We can find the area of the "slice of pie" or sector with a central angle θ using proportions. Once again we use the ratio of the angle θ to the whole circle which is $2\pi\ radians$. This is set equal to the ratio of the area of the sector to the area of the circle and simplify.

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

$$\boldsymbol{A = \frac{1}{2} r^2 \theta}$$

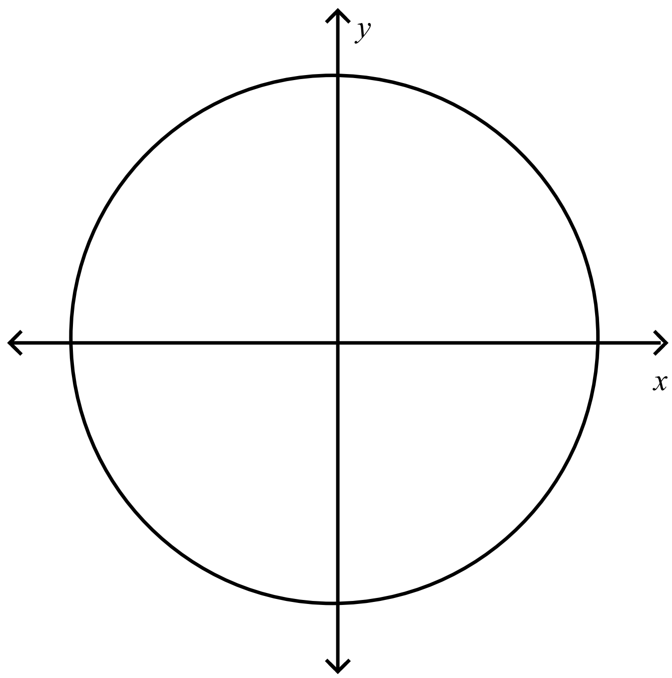
(θ is in radians)

Find the area of the sector of a circle of radius 2 feet formed by an angle of 30°. Round the answer to two decimal places.

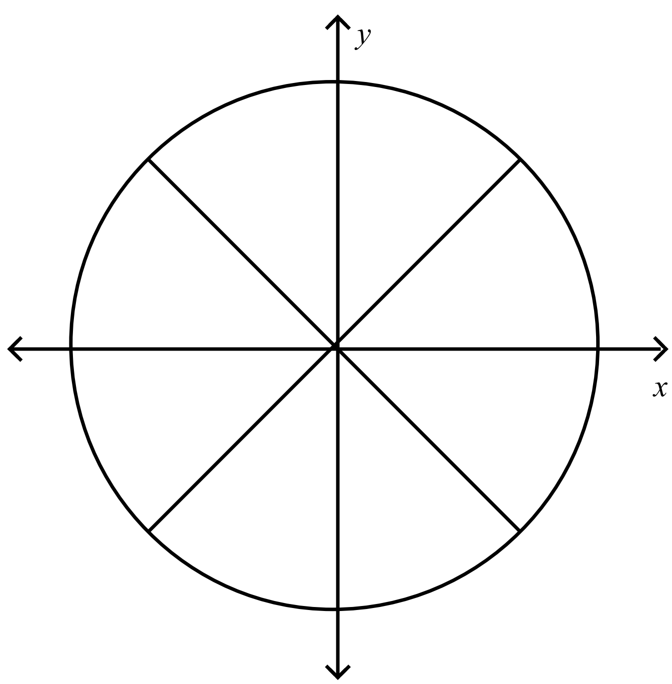
Lesson 6.2

Determine what the angle measure is in both degree and radians:

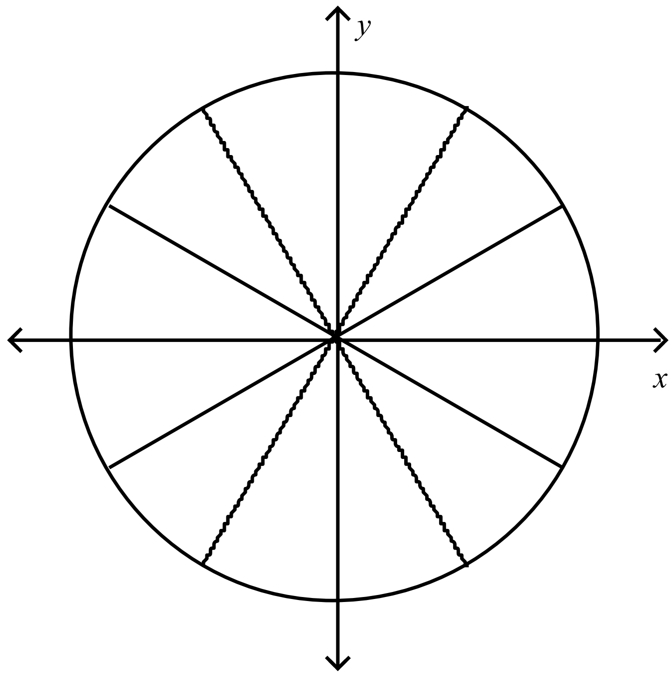
The Unit Circle



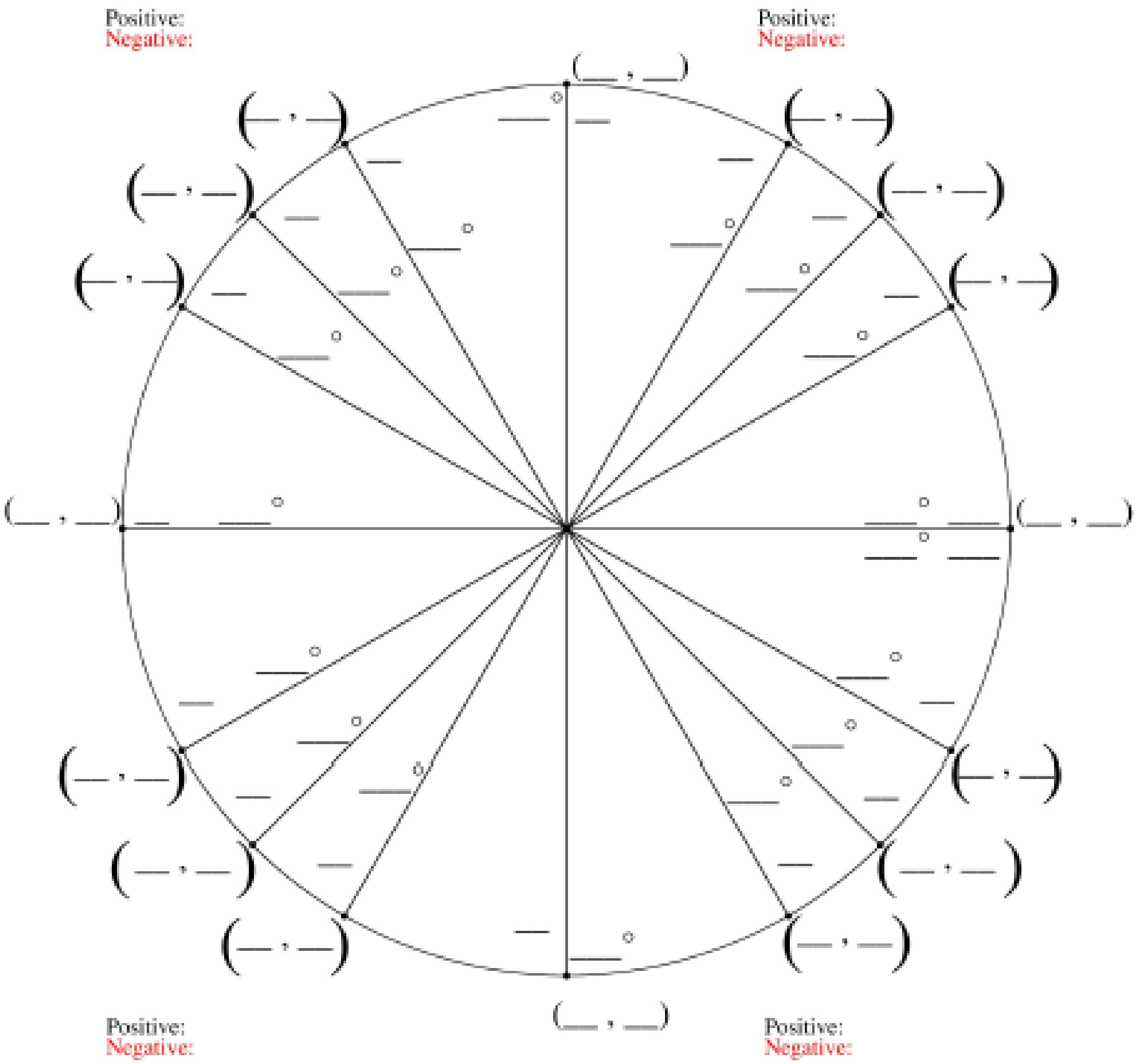
Unit Circle $\frac{\pi}{4}$ reference number



Unit Circle $\frac{\pi}{3}$ and $\frac{\pi}{6}$ reference number



Fill in The Unit Circle



EmbeddedMath.com

Summary (pg. 361): Y*ou need to know these special angle/radian relationships!!*

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π