Precalculus

Lesson 6.3: Properties of the Trigonometric Functions

Mrs. Snow, Instructor

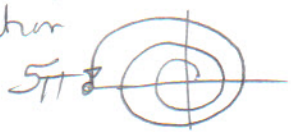
$$\frac{\sin(\text{gerine})}{\cos(\text{gerine})} =$$

Finding the exact value of a trig function and graph

(a) $\sin\frac{17\pi}{4}$ $(\frac{8\pi}{4}$ in 1 revolution) $= \frac{\sqrt{2}}{2}$

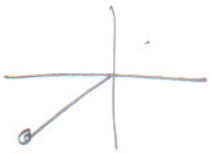
$\frac{17\pi}{4}$

(b) $\cos(5\pi)$ 2π in 1 revolution $= -1$



5π

(c) $\tan\frac{5\pi}{4} = -1$

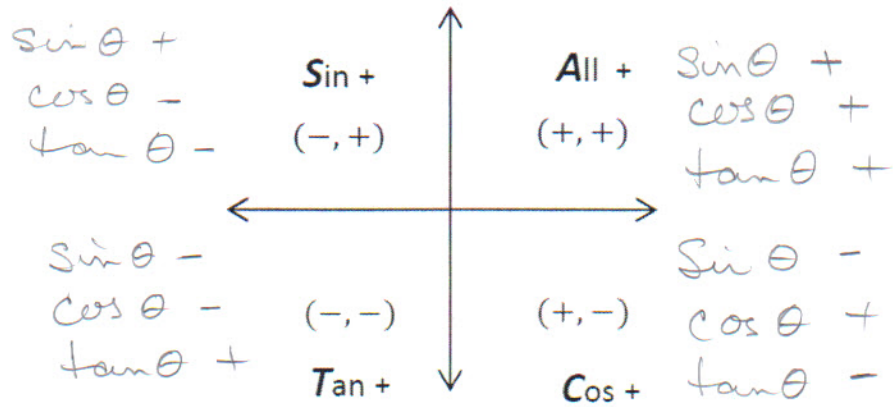


SIGNS OF THE TRIGONMETRIC FUNCTIONS

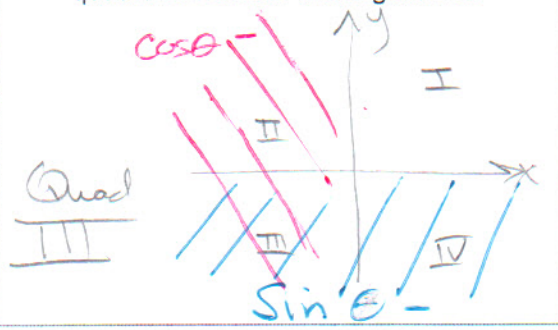
Looking at the Cartesian Plane remember the signs of x and y in each of the quadrants. Superimpose this with the location of **t,** and you can determine the appropriate sign for each trig function.

A little mnemonic to remember which trig function is positive in which quadrant is:

All ***S***tudents ***T***ake ***C***alculus



If $\sin\theta < 0$ and $\cos\theta < 0$, name the quadrant in which the angle θ lies.

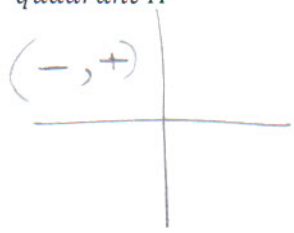


Find the sign of the expression if the terminal point is determined by t in the given quadrant.

$\cos t \cdot \sin t, \quad quadrant\ II$

(-)(+)

= −



Fundamental Trigonometric Identities

Trigonometric Identities –In mathematics, trigonometric identities are equalities that involve trigonometric functions and are true for every single value of the occurring variables. The relationship between our basic trig functions and their reciprocals are the **Reciprocal Identities** we also need to know another 3 important identities know as

OR $\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$

Reciprocal Identities

$$csc\theta = \frac{1}{sin\theta} \qquad sec\theta = \frac{1}{cos\theta} \qquad cot\theta = \frac{1}{tan\theta}$$

$\tan\theta = \frac{1}{\cot\theta}$

Quotient Identites

$$tan\theta = \frac{sin\theta}{cos\theta} \qquad cot\theta = \frac{cos\theta}{sin\theta}$$

Memorize!

Pythagorean Identities

$$sin^2\theta + cos^2\theta = 1 \qquad tan^2\theta + 1 = sec^2\theta \qquad 1 + cot^2\theta = csc^2\theta$$

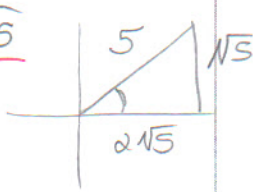
Finding exact values using identities

Given $\sin\theta = \frac{\sqrt{5}}{5}$ *and* $\cos\theta = \frac{2\sqrt{5}}{5}$, find the exact values of the four remaining trig functions of θ using identities.

Sine & cosine both + Quad I

$\sin\theta = \frac{O}{H} = \frac{\sqrt{5}}{5} = \frac{y}{h}$ $\quad \csc\theta = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\cos\theta = \frac{A}{H} = \frac{2\sqrt{5}}{5} = \frac{x}{h}$ $\quad \sec\theta = \frac{5}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{2}$



$\tan\theta = \frac{O}{A} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$ $\quad \cot\theta = 2$

Find the exact value of each expression. Do not use a calculator

a. $\tan 20° - \frac{\sin 20°}{\cos 20°}$

↑ Quotient ID

$\tan 20 - \tan 20°$

$= 0$

b. $\sin^2 \frac{\pi}{12} + \frac{1}{\sec^2 \frac{\pi}{12}}$ → $\cos^2\theta = \frac{1}{\sec^2\theta}$

$\sin^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{12} = 1$

Pythagorean ID.

+ Q I & II - Q II & III

Given that $\sin\theta = \frac{1}{3}$ and $\cos\theta < 0$, find the exact value of each of the remaining five trig functions.

+ −

$\sin\theta = \frac{O}{H} = \frac{1}{3} = \frac{y}{h}$

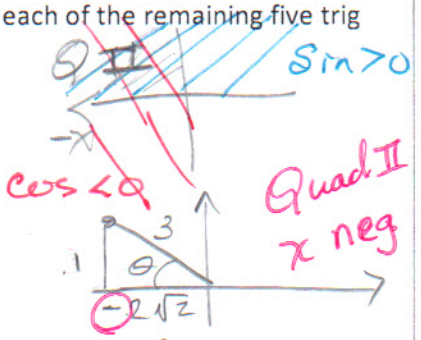
$\cos\theta = \frac{A}{H} = \frac{-2\sqrt{2}}{3}$

$\tan\theta = \frac{O}{A} = \frac{1}{-2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{4}$

$\csc\theta = 3$

$\sec\theta = -\frac{3}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{-3\sqrt{2}}{4}$

$\cot\theta = -2\sqrt{2}$



$x^2 + 1 = 9$

$x^2 = 8$

$x = -2\sqrt{2}$

Given that $\tan\theta = \frac{1}{2}$ and $\sin < 0$, find the exact value of each of the remaining five trig functions.

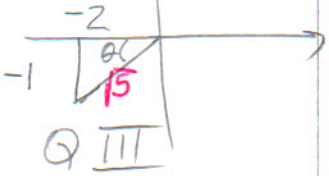
+ −

$\tan\theta = \frac{O}{A} = \frac{1}{2}$ $\cot\theta = 2$

$\sin\theta = \frac{-1}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$ $\csc\theta = -\sqrt{5}$

$\cos\theta = \frac{-2}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$

QIII x & y neg.



$4 + 1 = h^2$

$5 = h^2$ $\sqrt{5} = h$

Even-Odd Properties of Trigonometric Functions

A function is considered even if

- $f(-\theta) = f(\theta)$ for all θ in the domain of the function.

The function is odd if

- $f(-\theta) = -f(\theta)$ for all θ in the domain of the function.

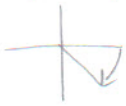
Even-Odd Properties

EVEN

odd $\begin{cases} \sin(-\theta) = -\sin\theta \\ \csc(-\theta) = -\csc\theta \end{cases}$ $\cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta$ $\begin{cases} \tan(-\theta) = -\tan\theta \\ \cot(-\theta) = -\cot\theta \end{cases}$ odd

Find the exact values of:

a) $\sin(-45°) = -\sin 45 = -\frac{\sqrt{2}}{2}$



b) $\cos(-\pi) = \cos\pi = -1$

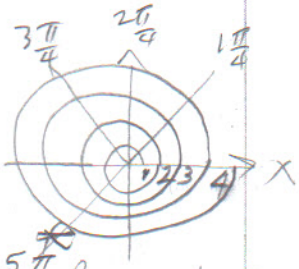
c) $\cot\left(-\frac{3\pi}{2}\right) = -\cot\frac{3\pi}{2} = 0$

x
y

d) $\tan\left(-\frac{37\pi}{4}\right) = -\tan\frac{37\pi}{4} = -\tan\left(\frac{32\pi}{4} + \frac{5\pi}{4}\right)$

$= -\tan\frac{5\pi}{4} = -1$

1 revolution = $\frac{8\pi}{4}$



$8\overline{)37}$ How many revolutions?

32

5R 4 w/ $\frac{5\pi}{4}$ Remainding!