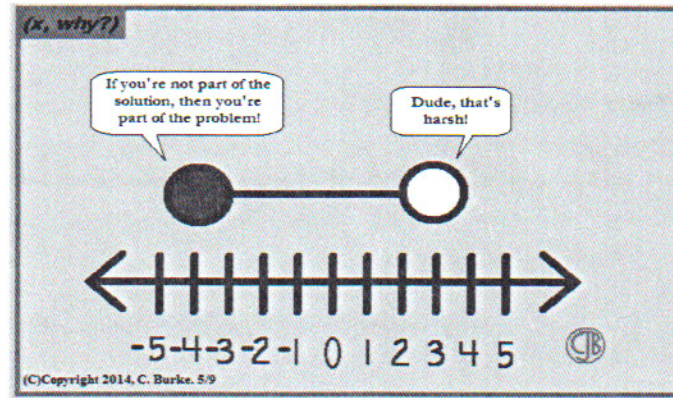


Precalculus  
Lesson 4.6: Polynomial and Rational Inequalities  
Mrs. Snow, Instructor



This section covers the processes to graph inequalities of polynomials and rational functions

**Solution**

1. Write the inequality so that a polynomial/rational expression is on the left side and 0 is on the right side
2. Determine the real zeros (x-intercepts) of  $f$  and any real numbers for which the expression is undefined.
3. Using the zeros and undefined values, divide the real number line into intervals
  - a. Is the inequality  $<$ ,  $>$ ,  $\leq$ , or  $\geq$  at zero?
  - b. Equality means a point on the zero
  - c. Not equal means a circle
4. Select a number in each interval, evaluate at that number. Focus on the sign of the factors and the overall outcome of  $\pm$ . Don't worry about the exact numerical answer.

Solve the inequalities algebraically and graph the solution

$$x^4 > x$$

$$x^4 - x > 0$$

①  $x(x^3 - 1) = 0$  \*

$$(x)(x-1)(x^2+x+1) = 0$$

treat as equality to find intervals

②

$$x = 0 \quad x = 1$$

$$x^2 + x + 1 = 0$$

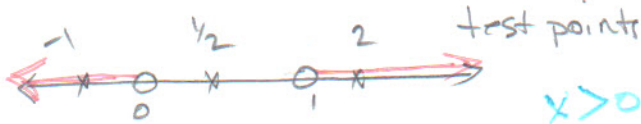
no real sol.

$$b^2 - 4ac$$

$$1 - 4(1)(1) = -3$$

discriminant negative

③



$x > 0$  Intervals have circles

Factors

$x$	-	+	+
$x-1$	-	-	+
$x^2+x+1$	+	+	+
	+	-	+

Sign of factor using test points

← Sign of function

Intervals  $> 0$  are solutions

Ans  $(-\infty, 0) \cup (1, \infty)$

\* Know Sum & difference of two cubes factoring formula!!

$$\frac{4x+5}{x+2} \geq 3$$

$$\frac{4x+5}{x+2} - 3 \geq 0 \quad \text{common denom.}$$

$$\frac{4x+5}{x+2} - 3 \frac{(x+2)}{(x+2)} \geq 0$$

$$\frac{4x+5-3x-6}{x+2} \geq 0$$

$$\frac{x-1}{x+2} \geq 0$$

$$\begin{aligned} x-1=0 & \quad x+2 \neq 0 \\ x=1 & \quad x \neq -2 \end{aligned}$$



$x-1$	-	-	+
$x+2$	-	+	+
	<u>+</u>	-	<u>+</u>

$$\underline{(-\infty, -2) \cup (1, \infty) \text{ Ans}}$$

Solutions  $\geq 0$  so intervals have dots.  
Careful! Denominator has a discontinuity so value has a circle 0

Remember if inequality had been  $\leq$  we would look at negative sign interval for the solution interval.

textbook pg. 240

### SUMMARY Steps for Solving Polynomial and Rational Inequalities Algebraically

**STEP 1:** Write the inequality so that a polynomial or rational expression  $f$  is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \quad f(x) \geq 0 \quad f(x) < 0 \quad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient and find the domain of  $f$ .

**STEP 2:** Determine the real numbers at which the expression  $f$  equals zero and, if the expression is rational, the real numbers at which the expression  $f$  is undefined.

**STEP 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

**STEP 4:** Select a number in each interval and evaluate  $f$  at the number.

(a) If the value of  $f$  is positive, then  $f(x) > 0$  for all numbers  $x$  in the interval.

(b) If the value of  $f$  is negative, then  $f(x) < 0$  for all numbers  $x$  in the interval.

If the inequality is not strict ( $\geq$  or  $\leq$ ), include the solutions of  $f(x) = 0$  that are in the domain of  $f$  in the solution set. Be careful to exclude values of  $x$  where  $f$  is undefined.