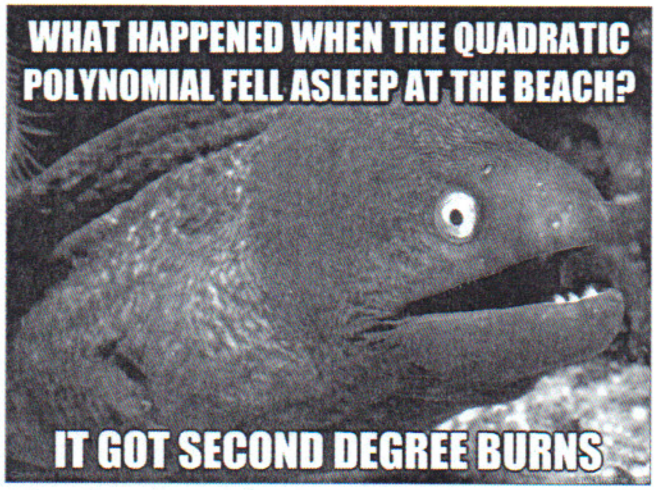
Precalculus

Lesson 4.1 Polynomial Functions and Models

Mrs. Snow, Instructor



Let's review the definition of a polynomial.

A polynomial function of degree n is a function of the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer and

- The numbers $a_0, a_1, a_2, \ldots a_n$ are called coefficients of the polynomial.
- The number a_0 is the constant term.
- The number a_n, the coefficient of the highest power, is the leading coefficient.
- The **degree** of the polynomial function is the largest power of *x* that appears.

Identify the polynomial functions, state degree.

$f(x) = 3x - 4x^3 + x^8$

polyn deg = 8

$g(x) = \dfrac{x^2 + 3}{x - 1}$ NO

$h(x) = 5x^0 = 5$

polyn deg 0

$F(x) = (x - 3)(x + 2)$ polyn

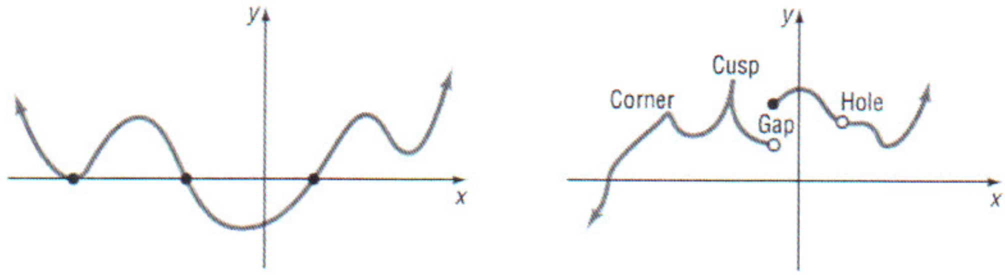
$x^2 - x - 6$ deg 2

$G(x) = 3x - 4x^{-1}$

No (neg exponent)

$H(x) = \frac{1}{2}x^3 - \frac{2}{3}x^2 + \frac{1}{4}x$

polyn deg 3



(a) Graph of a polynomial function: smooth, continuous

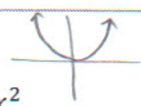
(b) Cannot be the graph of a polynomial function

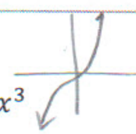
The textbook defines a **power function** as a monomial function (a single termed polynomial).

$$f(x) = ax^n$$
$$a \text{ is a real nonzero number, and } n > 0$$

$f(x) = 3x$	$f(x) = -5x^2$	$f(x) = 8x^3$	$f(x) = -5x^4$
degree 1	degree 2	degree 3	degree 4

Common power functions





$y = x$ $\quad$ $y = x^2$ $\quad$ $y = x^3$ $\quad$ $y = x^4$ $\quad$ $y = x^5$

1. For even degree power functions, the graph is symmetric with respect to the y-axis so it is considered even.

2. Odd degree power functions, the graph is symmetric with respect to the origin, so they are considered odd.

3. What three points will the even functions always have? $(-1, 1)$ $(1, 1)$ & $(0, 0)$

4. What three points will the odd functions always have? $(1, 1)$ $(0, 0)$ $(-1, -1)$

5. Notice that the exponent increases in magnitude, the graph increases more rapidly. For super small x values, x near the origin, the graph tends to flatten out and lie closer to the x-axis.

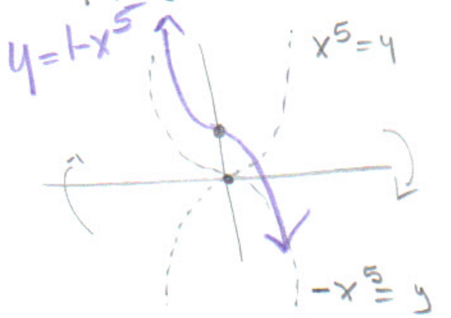
6. As x heads off to positive ~~of~~ or negative infinity, Power Functions help us to describe the end behavior of a polynomial functions.

Graphing a polynomial function using transformations:

- State the transformations
- Sketch the graphs of the following functions,

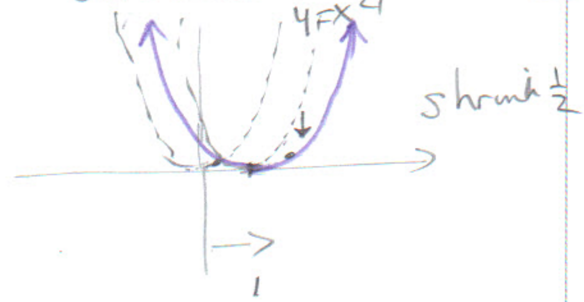
$f(x) = 1 - x^5$ $\quad x^5$ parent

- reflect over x-axis
- ↑1 vertical slide

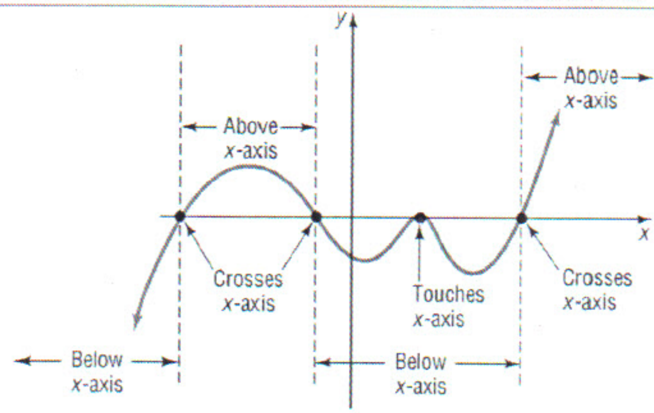


$f(x) = \frac{1}{2}(x-1)^4$ $\quad x^4$ parent

- Horiz → Rt. 1 unit
- Vertical shrink factor $\frac{1}{2}$



Zeros and Multiplicities



When we look for the **zeros** of a polynomial equation, we are looking for those values of **x** that are solutions to the equation or $P(x) = 0$. Graphically, we see the zeros where the graph crosses or touches the x-axis.

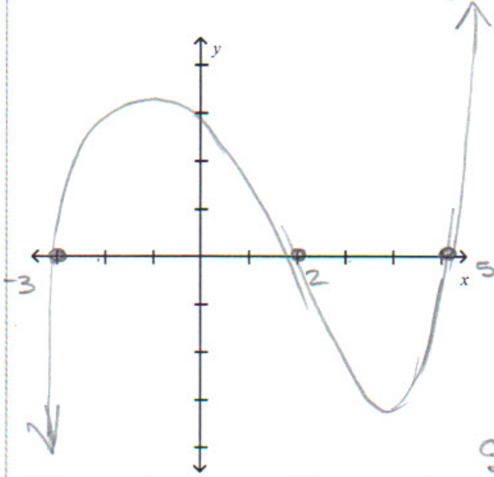
Real Zeros of Polynomials

- If f is a polynomial and **r** is a real number for which $f(r) = 0$, then the following are equivalent:
  - ➢ r is a zero of f.
  - ➢ r is an x-intercept of the graph of f.
  - ➢ r is a solution of the equation $f(x) = 0$.
  - ➢ $(x - r)$ is a factor of $f(x)$.

Using Zeros to Graph a Polynomial Function

Sketch Zeros to graph a Polynomial Function of degree 3. $\longrightarrow$

$(zeros = -3, 2, and\ 5)$



approx.!

we get zeros from factors
go backwards!

What are the factors of this graph? $x = -3$ $\quad x = 2$ $\quad x = 5$

factor: $x+3=0$ $\quad x-2=0$ $\quad x-5=0$

If asked what is the polynomial function? (Yes! multiply out the factors)

$(x+3)(x-2)(x-5) = f(x)$

$= x^3 - 4x^2 - 11x + 30 = f(x) \leftarrow$ polynomial.

Remember you can also take a polynomial function, factor it and then graph. To make this process easier, always remember to look for common factors of each term to factor out.

MULTIPLICITIES

Given the factor $(x - r)$: If $(\boldsymbol{x} - \boldsymbol{r})$ occurs more than once, **r** is called a **repeated or multiple,** ~~multiple~~, zero of f.

$(x - r)^m$

Or, **r** is a **zero with a multiplicity of *m***

m tells us how many times r is a zero

Identify the zeros and their multiplicities.

$$P(x) = x^4(x-2)^3(x+1)^2$$

x^4	$(x-2)^3$	$(x+1)^2$	
$x=0$	$x=2$	$x=-1$	$\leftarrow$ zeros
$m=4$	$m=3$	$m=2$	$\leftarrow$ multiplicity

Graphing:

If the zero is a real number, then it will be an x-intercept.

- Multiplicity of a zero is EVEN → graph will TOUCH the x-axis at *r*
- Multiplicity of a zero is ODD → graph will CROSS the x-axis at *r*
- The higher the multiplicity the <u>flatter</u> the graph at the zero

Turning points:

If $f(x)$ has a **degree of n**, then the graph of f has at most $\boldsymbol{n-1}$ *local extrema*.

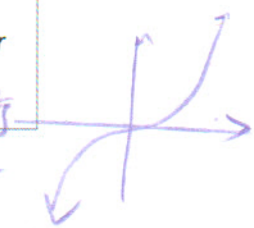
* If $f(x)$ has a **degree of n**, then the graph of f has at most $\boldsymbol{n-1}$ *turning points*

Turn this around: If a polynomial function f has $n-1$ turning points, the degree of f is at least n.

example

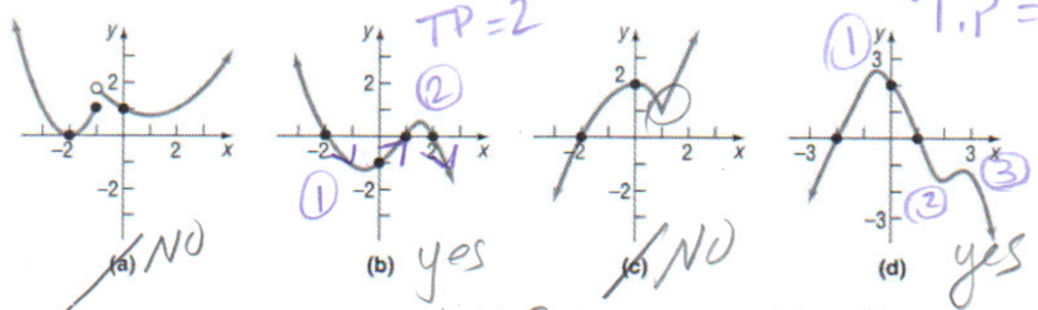
at most," be careful with this term. A polynomial of degree 5 will have at most 4 extrema or at most 4 turning points. IT MAY NOT HAVE 4 TURNING POINTS OR EXTREMA! Why???

rules say at most $f(x) = x^5$ no turning pts no extrema



From our last chapter remember our local maximums and minimums; they also known as **extrema** of a polynomial. These are the "hills" or "valleys" where the graph changes from increasing to decreasing or vice versa. ***An extrema is a y-value, not a point.***

Which of the graphs in Figure 16 could be the graph of a polynomial function? For those that could, list the zeros and state the least degree the polynomial can have. For those that could not, say why not.



(b) $x=-2, x=1, x=2$
2 turning pts
$n-1=2$
$n=3$ degree

(d) $x=-2$ $x=1$
3 T.P.
$n-1=3$
$n=4=$degree

End Behavior

When we graph these polynomials, we put arrows on the end of the curve to show that the graph continues on to infinity. What is happening to the end of the graph? Is the graph rising (increasing) or falling (decreasing)? The **end behavior** of a polynomial is the description of what happens as x approaches infinity (the positive direction) and approaches negative infinity (the negative direction). We have a certain notation use to describe the end behavior.

For large values of x, either positive or negative, that is for large $|x|$ the graph of the polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

resembles the graph of the power function

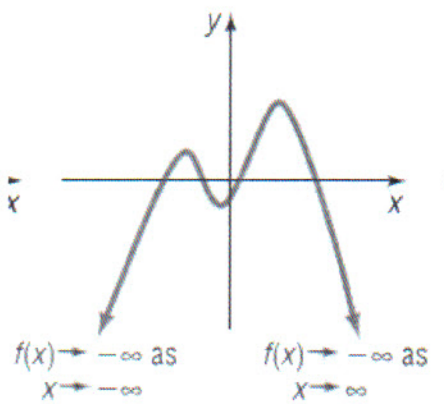
$$f(x) = ax^n$$

$x \to \infty$ means as x goes to infinity	$x \to -\infty$ means as x goes to negative infinity

For polynomials with degree ≥ 2 *and even*

LEADING COEFFICIENT IS NEGATIVE

The graph falls both to the left and right.

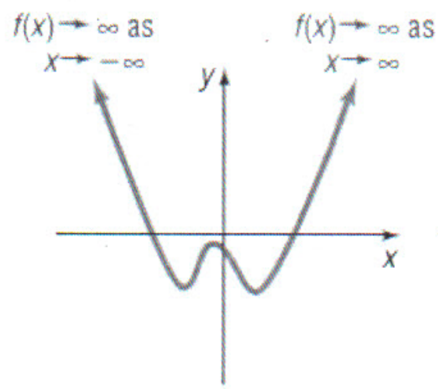


(b)

$n \geq 2$ even; $a_n < 0$

LEADING COEFFICIENT IS POSITIVE

The graph rises both to the left and right.



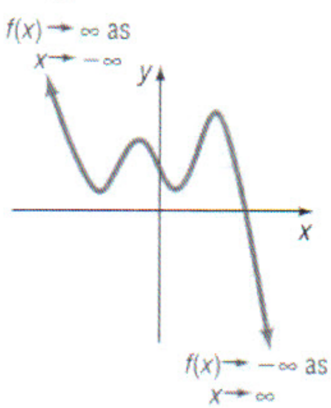
(a)

$n \geq 2$ even; $a_n > 0$

For polynomials with degree ≥ 1 *and odd*

LEADING COEFFICIENT IS NEGATIVE

The graph of f rises to the left and falls to the right.

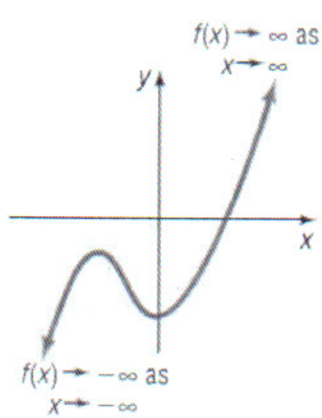


(d)

$n \geq 3$ odd; $a_n < 0$

LEADING COEFFICIENT IS POSITIVE

The graph of f falls to the left and rises to the right.



(c)

$n \geq 3$ odd; $a_n > 0$

Analyze a Graph of a Polynomial Function

Follow the steps to analyze and then graph a polynomial function:

Analyze the graph of the polynomial function:

$$f(x) = (2x + 1)(x - 3)^2$$

$= (2x+1)(x^2-6x+9)$

$= 2x^3-12x^2+18x$

$\quad\quad x^2-6x+9$

$= 2x^3-11x^2+12x+9$

1. Determine the end behavior of the graph of the function

3rd degree, +LC

2. Find the x- and y-intercepts of the graph of the function

$2x+1=0 \quad x-3=0$

$x=-\frac{1}{2} \quad x=3 \quad y=9$

3. Determine the zeros of the function and their multiplicity.

$x=-\frac{1}{2} \quad x=3$

$m=1 \quad m=2$

cross touches

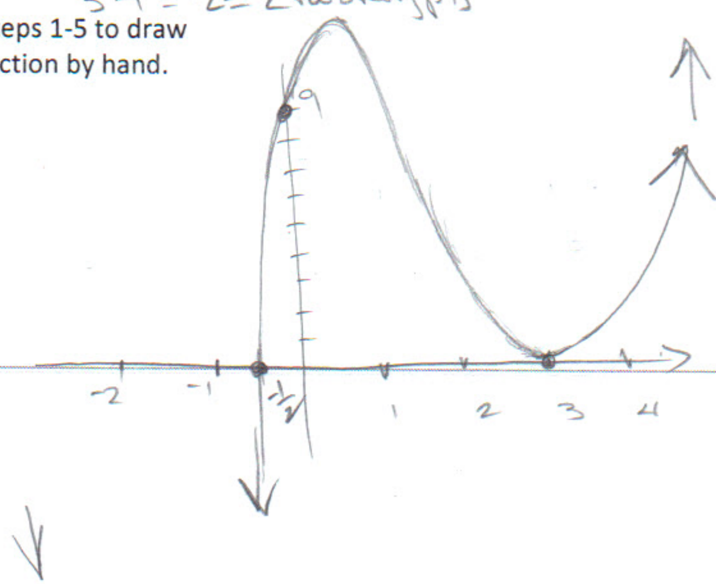
4. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

5. Determine the maximum number of turning points on the graph of the function.

degree = 3 = n $\quad$ n − 1 = tp

3 − 1 = 2 = 2 turning pts

6. Use the information in Steps 1-5 to draw a complete graph of the function by hand.



Analyze the graph of the polynomial function:

$f(x) = x^2(x-4)(x+1) \rightarrow x^2(x^2-3x-4)$

$f(x) = x^4 - 3x^3 - 4x^2$

Zeros:

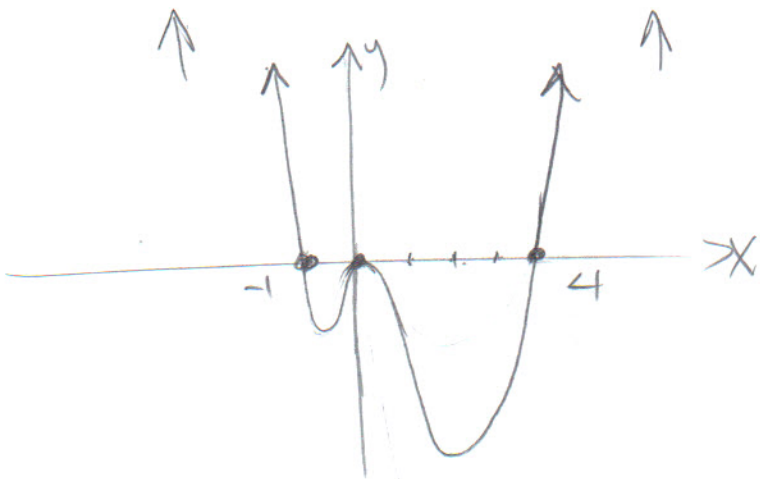
$x=0$	$x=4$	$x=-1$	y-incpt $x=0, y=0$
$m=2$	$m=1$	$m=1$	
touch	cross	cross	

end behav

x^4 even

$n-1 = $ t point

$4-1 = 3$ t points



Pg 189

SUMMARY

Graph of a Polynomial Function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \qquad a_n \neq 0$

Degree of the polynomial function f: n
Graph is smooth and continuous.
Maximum number of turning points: $n - 1$
At a zero of even multiplicity: The graph of f touches the x-axis.
At a zero of odd multiplicity: The graph of f crosses the x-axis.
Between zeros, the graph of f is either above or below the x-axis.
End behavior: For large $|x|$, the graph of f behaves like the graph of $y = a_n x^n$.

Pg. 190

SUMMARY Analyzing the Graph of a Polynomial Function

STEP 1: Determine the end behavior of the graph of the function.

STEP 2: Find the x- and y-intercepts of the graph of the function.

STEP 3: Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

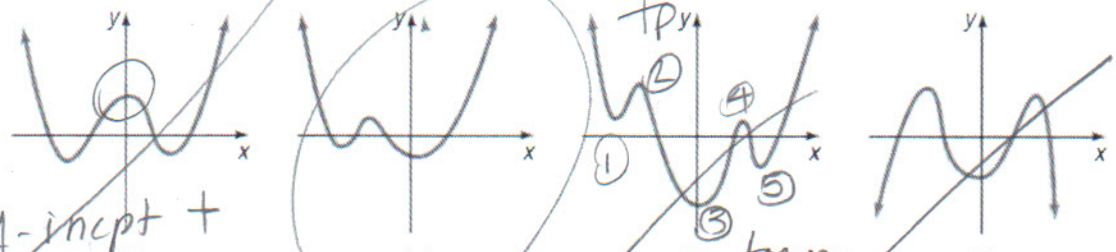
STEP 4: Determine the maximum number of turning points on the graph of the function.

STEP 5: Determine the behavior of the graph near each x-intercept.

STEP 6: Use the information in Steps 1 through 5 to draw a complete graph of the function.

Which graph could model the polynomial and why are the other graphs eliminated?

$$f(x) = x^4 + ax^3 + bx^2 - 5x - 6$$



(a) (b) (c) (d)

y-incpt +

too many

turn pt = 5

X^4 ↑↗

$4-1 = 3$ turn pt

y-incpt
$y=-6$

b) ↑↗

3 t.p.

y-incpt neg