## Precalculus

## Lesson 4.6: Polynomial and Rational Inequalities

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This section covers the processes to graph inequalities of polynomials and rational functions

## Solution

1. Write the inequality so that a polynomial/rational expression is on the left side and 0 is on the right side
2. Determine the real zeros (x-intercepts) of $f$ and any real numbers for which the expression is undefined.
3. Using the zeros and undefined values, divide the real number line into intervals
a. Is the inequality $<,>, \leq$, or $\geq$ at zero?
b. Equality means a point on the zero
c. Not equal means a circle
4. Select a number in each interval, evaluate at that number. Focus on the sign of the factors and the overall outcome of $\pm$. Don't worry about the exact numerical answer.

Solve the inequalities algebraically and graph the solution
$x^{4}>x$

$$
\frac{4 x+5}{x+2} \geq 3
$$

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## SUMMARY Steps for Solving Polynomial and Rational Inequalities Algebraically

Step 1: Write the inequality so that a polynomial or rational expression $f$ is on the left side and zero is on the right side in one of the following forms:

$$
f(x)>0 \quad f(x) \geq 0 \quad f(x)<0 \quad f(x) \leq 0
$$

For rational expressions, be sure that the left side is written as a single quotient and find the domain of $f$.
STEP 2: Determine the real numbers at which the expression $f$ equals zero and, if the expression is rational, the real numbers at which the expression $f$ is undefined.
Step 3: Use the numbers found in Step 2 to separate the real number line into intervals.
Step 4: Select a number in each interval and evaluate $f$ at the number.
(a) If the value of $f$ is positive, then $f(x)>0$ for all numbers $x$ in the interval.
(b) If the value of $f$ is negative, then $f(x)<0$ for all numbers $x$ in the interval.

If the inequality is not strict $(\geq$ or $\leq$ ), include the solutions of $f(x)=0$ that are in the domain of $f$ in the solution set. Be careful to exclude values of $x$ where $f$ is undefined.

