Precalculus

Lesson 4.2: The Real Zeros of a Polynomial Function

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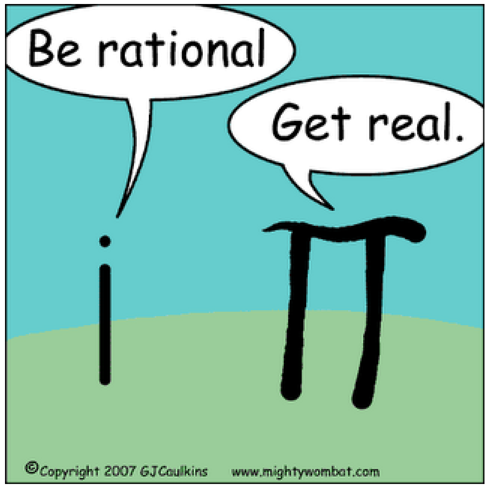

Dividing polynomials is a very similar process to the old long divisions we did in elementary school:

$$842 \div 15 = ???$$

Proper format for division of polynomials:

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad f(x) = q(x)g(x) + r(x)$$

$f(x)$: dividend, $q(x)$: quotient, $g(x)$: divisor, $r(x)$: remainder

Divide: $6x^2 - 26x + 12 \ \ by \ x - 4$	Divide using synthetic division; note this works only for divisors in the form of $\boldsymbol{x - c}$: $2x^3 - 7x^2 + 5 \quad by \quad x - 3$

The Remainder Theorem

If the polynomial f(x) is divided by $x - c$, then the remainder, $\boldsymbol{r(x)}$, is the value $f(c)$.

while we could use division to see if there is a remainder, our theorem says that the ***remainder*** $\boldsymbol{r(x) = f(c)}$

Using the Remainder Theorem, to find the remainder of:
$f(x) = x^3 - 4x^2 - 5$ (you can check your work with division).

1. $x - 3$, $\quad c =$ $\qquad\qquad$ $x + 2$, $\quad c =$

Let's take the remainder theorem a step farther. If $r(x) = 0$, what does that tell us about the factor, $(x - c)$??

Factor Theorem

$$Let\ f\ be\ a\ polynomial\ function. \quad (x - c)\ is\ a\ factor\ of\ f(x)\ if\ and\ only\ if\ f(c) = 0$$

When $f(c) = 0$, the remainder is 0, therefore, $(x - c)$ is a factor

Use the factor theorem to determine whether the function

$$f(x) = 2x^3 - x^2 + 2x - 3$$

has the factor

a) $x - 1\,, \quad c =$

b) $x + 2\,, \quad c =$

Finding the roots

Take a look at $P(x) = (x - 2)(x - 3)(x + 4)$ multiplying the factors together we get:

$P(x) = x^3 - x^2 - 14x + 24$ Where did the constant 24 come from?

So, the constants of the factors multiplied out give us the constant of P(x). If the product of the zeros equals the constant then the zeros are all factors of the constant! We use this fact in the form of the **Rational Zeros Theorem**.

Rational Zeros Theorem

If the polynomial, P, has integer coefficients,

then every rational zero of P is of the form $\pm\frac{\boldsymbol{p}}{\boldsymbol{q}}$

q= is a factor of the leading coefficient

$$f(x) = \mathbf{3}x^3 + 8x^2 - 7x - \mathbf{12}$$

p= is a factor of the constant

After finding all the possible rational roots, one simply uses the factor theorem to determine if the number is in fact a root. Graphing calculators make this process much easier. Remember! If you are not allowed to use a graphing calculator, you will want to plug all values into the polynomial to determine which are roots.

List the potential rational zeros of:

$$f(x) = 3x^3 + 8x^2 - 7x - 12$$

From your textbook, pg 203

SUMMARY Steps for Finding the Real Zeros of a Polynomial Function

STEP 1: Use the degree of the polynomial to determine the maximum number of real zeros.

STEP 2: (a) If the polynomial has integer coefficients, use the Rational Zeros Theorem to identify those rational numbers that potentially could be zeros.

(b) Use substitution, synthetic division, or long division to test each potential rational zero. Each time that a zero (and thus a factor) is found, repeat Step 2 on the depressed equation.

In attempting to find the zeros, remember to use (if possible) the factoring techniques that you already know (special products, factoring by grouping, and so on).

NUMBER OF REAL ZEROS

A polynomial function cannot have more real zeros than its degree.

Solve by factoring: $x^2 + 13x + 36=0$	$2x^2 - 5x - 7 = 0$

1. List the possible rational zeros for the function below.
2. Using synthetic division, find the real zeros of the polynomial.
3. Factor completely.

$f(x) = 3x^3 + 8x^2 - 7x - 12$ you will find that: $(x = -1)$

For the following function:
List all possible rational roots
Find the real zeros of f(x) and write in factored form:

$f(x) = 2x^4 + 13x^3 + 29x^2 + 27x + 9$ $\qquad$ $x = -1, -3$

Factor by grouping to find the zeros of the function (no calculator!!!):

$$f(x) = 4x^3 - 32x^2 - x + 8$$