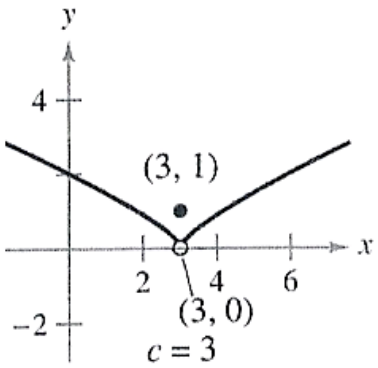
# Continuity & One-Sided Limits Homework

Use the graph to determine the limit, and discuss the continuity of the function.

(a) $\lim_{x\to c^+} f(x)$ (b) $\lim_{x\to c^-} f(x)$ (c) $\lim_{x\to c} f(x)$

3.

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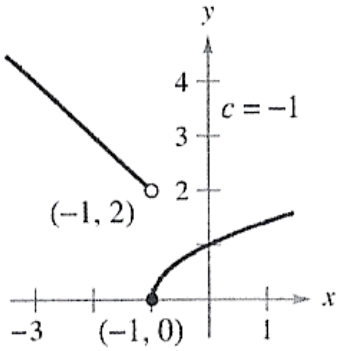



Find the limit (if it exists). If it does not exist, explain why.

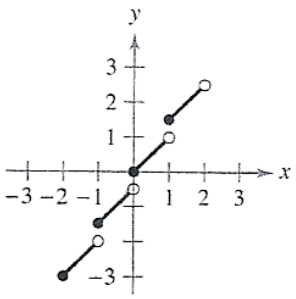
7. $\lim_{x\to 5^+} \dfrac{x-5}{x^2-25}$ 9. $\lim_{x\to -3^-} \dfrac{x}{\sqrt{x^2-9}}$ 12. $\lim_{x\to 2^+} \dfrac{|x-2|}{x-2}$ 10. $\lim_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4}$

14. $\lim_{\Delta x\to 0^+} \dfrac{(x+\Delta x)^2+x+\Delta x-(x^2+x)}{\Delta x}$ 18. $\lim_{x\to 1^+} f(x)$, where $f(x)=\begin{cases} x, & x\le 1 \\ 1-x, & x>1 \end{cases}$

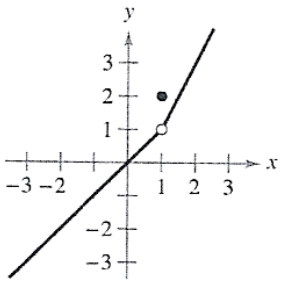
19. $\lim_{x\to \pi} \cot x$ 20. $\lim_{x\to \frac{\pi}{2}} \sec x$ 24. $\lim_{x\to 1} 1-\left[\!\left[-\frac{x}{2}\right]\!\right]$ 22. $\lim_{x\to 2^+} 2x-2$

Discuss the continuity of each function.

27. $f(x) = \frac{1}{2}[\![x]\!] + x$




28. $f(x)=\begin{cases} x, & x<1 \\ 2, & x=1 \\ 2x-1, & x>1 \end{cases}$



Find the x-values (if any) at which *f* is not continuous. Which of the discontinuities are removable?

35. $f(x) = 3x - \cos x$

40. $f(x) = \dfrac{x-3}{x^2-9}$

41. $f(x) = \dfrac{x+2}{x^2-3x-10}$

45. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

48. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

51. $f(x) = \csc 2x$

Find the constant *a*, or the constants *a* and *b*, such that the function is continuous on the entire real line.

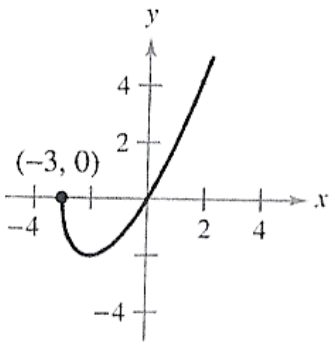
59. $f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$

60. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \ne a \\ 8, & x = a \end{cases}$

Describe the interval(s) on which the function is continuous.

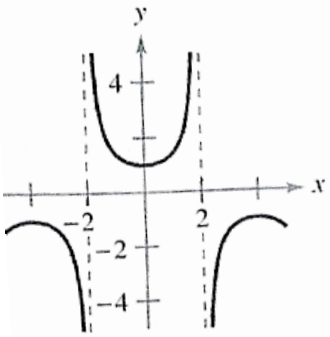
70.

$f(x) = x\sqrt{x+3}$




71.

$f(x) = \sec \dfrac{\pi x}{4}$




Discuss the continuity of the composite function $h(x) = f(g(x))$.

61. $f(x) = x^2$
$g(x) = x - 1$

63. $f(x) = \dfrac{1}{x-6}$
$g(x) = x^2 + 5$

Use the Intermediate Value Theorem and a graphing calculator to approximate the zero of the function in the interval $[0,1]$. Repeatedly "zoom in" on the graph of the function to approximate the zero accurate to two decimal places. Use the *zero* or *root* feature to approximate the zero accurate to four decimal places.

79. $f(x) = x^3 + x - 1$ 81. $g(t) = 2\cos t - 3t$

Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of *c* guaranteed by the theorem.

83. $f(x) = x^2 + x - 1, \quad [0,5], \quad f(c) = 11$

85. $f(x) = x^3 - x^2 + x - 2, \quad [0,3], \quad f(c) = 4$

86. $f(x) = \dfrac{x^2 + x}{x - 1}, \quad \left[\dfrac{5}{2}, 4\right] \quad f(c) = 6$

$$\lim_{\Delta x \to 0^+} \frac{(x + \Delta x)^2 + x + \Delta x - (x^2 + x)}{\Delta x}$$