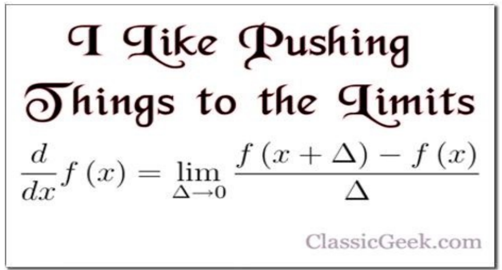
Calculus
Lesson 1.2-Finding Limits Graphically and Numerically
1.3 Finding Limits Analytically
Mrs. Snow, Instructor



From precalculus, we found several methods to get an idea of the behavior of the graph of f near an undefined value of x:

Estimating a Limit Numerically:

$$f(x) = \lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$$

x	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$							

Finding a Limit

$$f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

$$\lim_{x \to 2} f(x)$$

Limits That Fail to Exist:

Behavior that differs from right to left:

$$\lim_{x \to 0} \frac{|x|}{x}$$

Unbounded Behavior:

$$\lim_{x \to 0} \frac{1}{x^2}$$

Oscillating Behavior:

$$\lim_{x \to 0} \sin \frac{1}{x}$$

1.3 Finding Limits Analytically

We found that the limit of $f(x)$ as x approaches c does not depend on the value of f at $x = c$. It may happen, however that the limit is precisely $f(c)$. In these cases, the limit may be evaluated by **direct substitution**:

THEOREM 1.1 SOME BASIC LIMITS

Let b and c be real numbers and let n be a positive integer.

1. $\lim_{x \to c} b = b$ **2.** $\lim_{x \to c} x = c$ **3.** $\lim_{x \to c} x^n = c^n$

$a.\ \lim_{x \to 2} 3$ $b.\ \lim_{x \to -4} x$ $c.\ \lim_{x \to 2} x^2$

THEOREM 1.2 PROPERTIES OF LIMITS

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

The Limit of a Polynomial:

$$\lim_{x \to 2} \left(4x^2 + 3\right)$$

The **direct substitution** property is valid for all polynomial and rational functions with nonzero denominators:

THEOREM 1.3 LIMITS OF POLYNOMIAL AND RATIONAL FUNCTIONS

If p is a polynomial function and c is a real number, then

$$\lim_{x \to c} p(x) = p(c).$$

If r is a rational function given by $r(x) = p(x)/q(x)$ and c is a real number such that $q(c) \neq 0$, then

$$\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{q(c)}.$$

The Limit of a Rational Function

$$\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$$

THEOREM 1.4 THE LIMIT OF A FUNCTION INVOLVING A RADICAL

Let n be a positive integer. The following limit is valid for all c if n is odd, and is valid for $c > 0$ if n is even.

$$\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$$

THEOREM 1.5 THE LIMIT OF A COMPOSITE FUNCTION

If f and g are functions such that $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} f(x) = f(L)$, then

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L).$$

We see that the limits of many algebraic functions may be evaluated by direct substitution. The six basic trigonometric functions also exhibit a desirable quantity:

THEOREM 1.6 LIMITS OF TRIGONOMETRIC FUNCTIONS

Let c be a real number in the domain of the given trigonometric function.

1. $\lim_{x \to c} \sin x = \sin c$
2. $\lim_{x \to c} \cos x = \cos c$
3. $\lim_{x \to c} \tan x = \tan c$
4. $\lim_{x \to c} \cot x = \cot c$
5. $\lim_{x \to c} \sec x = \sec c$
6. $\lim_{x \to c} \csc x = \csc c$

$\lim_{x \to 0} \tan x$ $\qquad$ $\lim_{x \to \pi} x \cos x$ $\qquad$ $\lim_{x \to 0} \sin^2 x$

A Strategy for Finding Limits

We have reviewed several types of functions whose limits may be evaluated by direct substitution. This knowledge and Theorem 1.7 may be used to develop a strategy for finding limits:

THEOREM 1.7 FUNCTIONS THAT AGREE AT ALL BUT ONE POINT

Let c be a real number and let $f(x) = g(x)$ for all $x \neq c$ in an open interval containing c. If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ also exists and

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x).$$

Finding the limits of a Function:

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

Dividing Out Technique:

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Rationalizing Technique:

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$$

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$
2. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

A Limit Involving a Trigonometric Function:

$$\lim_{x \to 0} \frac{\tan x}{x}$$

$$\lim_{x \to 0} \frac{\sin 4x}{x}$$