

Precalculus
Lesson 10.3: The Ellipse
Mrs. Snow, Instructor

Ellipse: the collection, or locus, of all points in the plane, the sum of whose distances from two fixed points, called the foci, is constant.

Major axis it's the x-axis:

Equation of an Ellipse: Center at (0, 0); Major Axis along the x-Axis

An equation of the ellipse with center at (0, 0), foci at $(-c, 0)$ and $(c, 0)$, and vertices at $(-a, 0)$ and $(a, 0)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2 \quad (2)$$

Major axis is the y-axis:

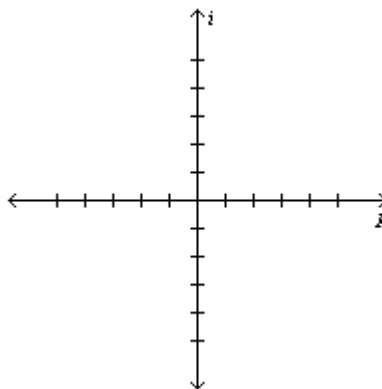
Equation of an Ellipse; Center at (0, 0); Major Axis along the y-Axis

An equation of the ellipse with center at (0, 0), foci at $(0, -c)$ and $(0, c)$, and vertices at $(0, -a)$ and $(0, a)$ is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2 \quad (3)$$

Finding an equation of an Ellipse

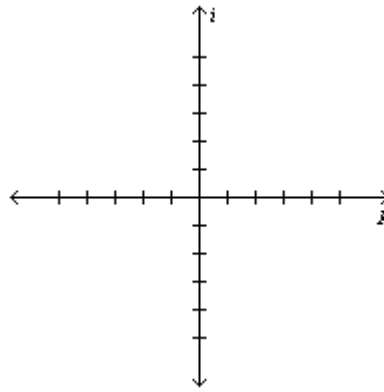
Find an equation of an ellipse with center at the origin, one focus at $(3, 0)$, and a vertex at $(-4, 0)$. Graph.



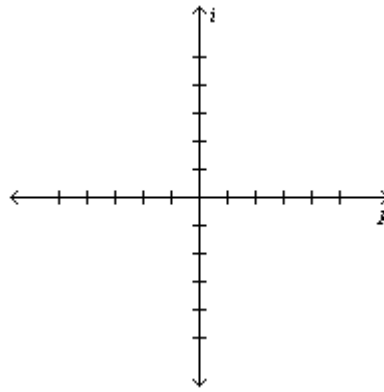
Analyze the Equation of an Ellipse

Analyze (find the center foci and vertices)

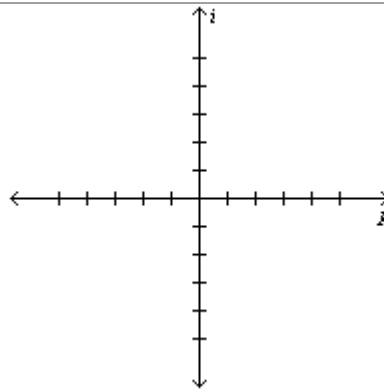
$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$



$$9x^2 + y^2 = 9$$



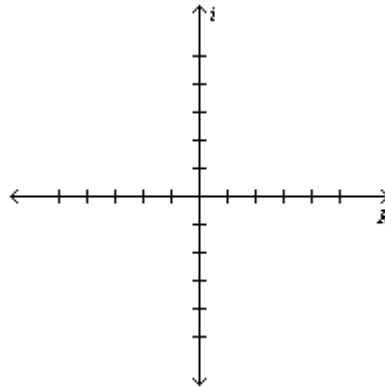
Find an equation of the ellipse having one focus at $(0, 2)$ and vertices at $(0, -3)$ and $(0, 3)$. Graph.



Ellipses with centers at (h, k)

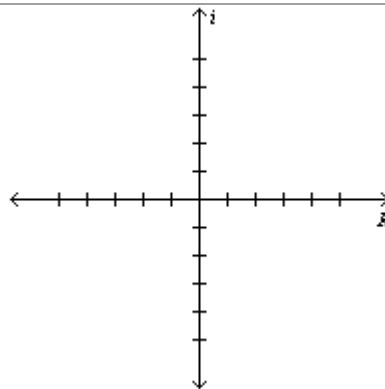
Equations of an Ellipse: Center at (h, k) ; Major Axis Parallel to a Coordinate Axis				
Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to the x -axis	$(h + c, k)$ $(h - c, k)$	$(h + a, k)$ $(h - a, k)$	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1,$ $a > b > 0$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to the y -axis	$(h, k + c)$ $(h, k - c)$	$(h, k + a)$ $(h, k - a)$	$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1,$ $a > b > 0$ and $b^2 = a^2 - c^2$

Find an equation of an ellipse with center at $(2, -3)$, one focus at $(3, -3)$, and one vertex at $(5, -3)$. Graph



Analyze the equation:

$$4x^2 + y^2 - 8x + 4y + 4 = 0$$



The whispering gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center?