Precalculus
Lesson 9.5: The Dot Product
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The concept of the dot product is used in calculus and in the applications of vectors in physics and engineering.

If $v = a_1 i + b_1 j = \langle a_1, b_1 \rangle$ and $w = a_2 i + b_2 j = \langle a_2, b_2 \rangle$ are vectors, then their dot product, denoted by $\boldsymbol{v \cdot w}$, is defined by

$$\boldsymbol{v \cdot w = a_1 a_2 + b_1 b_2}$$

say: "v dot w"

Given:

$$v = 2i - 3j \quad and \quad w = 5i + 3j$$

Find the following dot products:

a) $v \cdot w$

b) $w \cdot v$

c) $v \cdot v$

d) $w \cdot w$

e) $\|v\|$

f) $\|w\|$

The following properties of the Dot Product are useful in solving problems involving the Dot Product:

$$u \cdot v = v \cdot u$$

$$(au) \cdot v = a(u \cdot v) = u \cdot (av)$$

$$u \cdot (v + w) = u \cdot v + u \cdot w$$

$$v \cdot v = \|v\|^2$$

$$0 \cdot v = 0$$

The Dot Product Theorem

If we have u and v be vectors with initial points at the origin, the angle θ that is between u and v is $0 < \theta < \pi$.

$$\boldsymbol{u \cdot v = \|u\|\|v\| cos\theta}$$

$$\boldsymbol{cos\theta = \frac{u \cdot v}{\|u\|\|v\|}}$$

Find the angle θ between $u = 4i - 3j \quad and \quad v = 2i + 5j$

Orthogonal Vectors (a.k.a. perpendicular)

Two vectors $v \; and \; w$ are orthogonal, a.k.a. perpendicular, if and only if:

$$v \cdot w = 0$$

Determine whether the vectors pair are perpendicular

$$v = 2i - j \quad and \; w = 3i + 6j$$

Parallel Vectors

Two vectors $v \; and \; w$ are parallel if they are "multiples" of each other.

Determine whether the vectors in each pair are parallel.

$$v = 2i - j \quad and \; w = 6i - 3j$$

$$w = 3i + 4j \; and \quad r = 5i + 2j$$

Work

Work equals force times distance:

$$W = F \cdot D.$$

English units of force is pounds (lbs.)

When the force acting on the object is at an angle, remember to break it into its horizontal and vertical components.

A girl is pulling a wagon with a force of 50 pounds. How much work is done in moving the wagon 100 feet if the handle makes an angle of 30^o with the ground?