Precalculus
Lesson 5.2: One to One Functions; Inverse Functions
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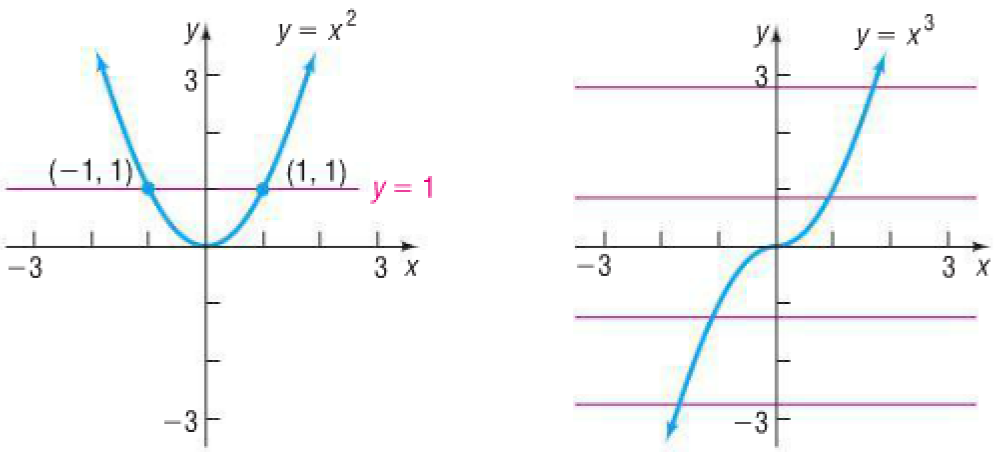
A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range. That is, if x_1 and x_2 are two different inputs of a function f, then f is one-to-one if $f(x_1) \neq f(x_2)$.

Translation. A function is one-to-one if every output is unique, there are no repeating outputs

Graphically we can determine a one-to-one relationship by using the **horizontal-line-test** to determine of *f* is one-to-one. Basically, this is analogous to the vertical line test, only horizontal.

Horizontal-line Test

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.



(a) A horizontal line intersects the graph twice; *f* is not one-to-one

(b) Every horizontal line intersects the graph exactly once; *g* is one-to-one

Inverse function

If a function is one-to-one with a domain **D** and a range **R**, then the inverse function of f, is f^{-1} the inverse function will have a domain **R** and a range **D**

Domain of f = Range of f^{-1} Range of f = Domain of f^{-1}

and

$$f(g(x)) = g(f(x)) = x$$

Inverses: An inverse function is a function that undoes the action of another function. Another way of saying inverse is opposite. Did you ever play "opposite day" with your parents? If you remember you probably would not confess it; Yes means No and No means Yes! Mathematically: x is y and y is x.

Find the inverse of the following one-to-one function:

$$\{(-3, -27), (-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)\}$$

(a) Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$.

(b) Verify that the inverse of $f(x) = 2x + 3$ is $f^{-1}(x) = \frac{1}{2}(x - 3)$

How to Find the Inverse Function

Find the inverse of $f(x) = 2x + 3$. Graph f and f^{-1} on the same coordinate axes

Step 1 Replace $f(x)$ with y. In $y = f(x)$, interchange the variables x and y to obtain $x = f(y)$. This equation defines the inverse function f^{-1} implicitly.

Step 2 If possible, solve the implicit equation for y in terms of x to obtain the explicit form of f^{-1}, $y = f^{-1}(x)$.

Step 3 Check the result by showing that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

The following function is one-to-one. Find its inverse and check the result.

$$f(x) = \frac{2x+1}{x-1}, x \neq 1$$

If a function is not one-to-one, it has no inverse function. However, by restricting the domain of that function, we can make the function 1-1 and find its inverse.

Find the inverse of $y = f(x) = x^2$ if $x \geq 0$. Graph f and f^{-1}.