Precalculus
Lesson 5.1: Composite Functions
Mrs. Snow, Instructor

Composite Functions: A composite function is a function that is made or composed of more than one "independent" function. In general, a number x is applied to one function the result or output is then applied to a second function.

Given two functions f and g, the **composite function,** denoted by $f \circ g$ (read as "f composed with g"), is defined by

$$\boxed{(f \circ g)(x) = f(g(x))}$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

The inner function output becomes the outer function input.

Domain of a composite:
The domain of a composite function, $f \ o \ g$, is defined whenever both $g(x) and\ f(g(x))$ are defined.

Evaluating a composite function

Suppose that $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find:

(a) $(f \circ g)(1)$ (b) $(g \circ f)(1)$ (c) $(f \circ f)(-2)$ (d) $(g \circ g)(-1)$

(a) $= f(g(1))$: $g(1) = 4(1) = 4$; $f(4) = 2(4^2) - 3 = 2(16) - 3 = 32 - 3 = \underline{\underline{29}}$

(b) $g(f(1))$: $f(1) = 2(1^2) - 3 = -1$; $g(-1) = 4(-1) = \underline{\underline{-4}}$

(c) $f(f(-2))$: $f(-2) = 2(-2)^2 - 3 = 8 - 3 = 5$; $f(5) = 2(5)^2 - 3 = 50 - 3 = \underline{\underline{47}}$

(d) $g(g(-1))$: $g(-1) = 4(-1) = -4$; $g(-4) = 4(-4) = \underline{\underline{-16}}$

Finding a composite function and its domain

Suppose that $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$.

Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.

$f(x)$ domain $= \mathbb{R}$ $\quad$ $g(x)$ domain $= \mathbb{R}$

$\therefore$ composite function Domain $\mathbb{R}$

(a) $f \circ g = f(g(x))$

$f(x) = x^2 + 3x - 1$

$f \circ g = (2x+3)^2 + 3(2x+3) - 1$

$= 4x^2 + 12x + 9 + 6x + 9 - 1$

$= 4x^2 + 18x + 17$

(b) $g \circ f = g(f(x))$

$g(x) = 2x + 3$

$g \circ f = 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x - 2 + 3$

$= 2x^2 + 6x + 1$

Finding the domain of the composite function $f(g(x))$

1. $g(x)$ must be defined so any x not in the domain of g must be excluded.
2. $f(g(x))$ must be defined so any x for which $g(x)$ is not in the domain of f must be excluded.

translation: *the domain of a composite function include restrictions for both the inner function and restrictions that show up in the composite function*

Find the domain of $(f \ o \ g)(x)$ when $f(x) = \frac{1}{x+2}$ *and* $g(x) = \frac{4}{x-1}$

$f(g(x))$

Restrictions $x \neq 1$ for $g(x)$ & $f \circ g(x)$

$f(x) = \frac{1}{x+2}$

$g(x)$

common denominator

$$f(g(x)) = \frac{1}{\frac{4}{x-1} + 2} = \frac{1}{\frac{4}{x-1} + \frac{2(x-1)}{(x-1)}}$$

complex fraction

$$= \frac{1}{\frac{4+2x-2}{x-1}} = \frac{1}{\frac{2x+2}{x-1}} \div = 1 \cdot \frac{x-1}{2x+2}$$

$f(g(x)) = \frac{x-1}{2x+2}$ & $x \neq -1$

Domain $f \circ g = \{x \mid x \neq 1, -1\} =$

or $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Show that two composite functions are equal

If $f(x) = 3x - 4$ and $g(x) = \frac{1}{3}(x + 4)$, show that

$$(f \circ g)(x) = (g \circ f)(x) = x$$

for every x in the domain of $f \circ g$ and $g \circ f$.

$(f \circ g)(x) =$

$\cancel{3}\left(\frac{1}{\cancel{3}}(x+\cancel{4})\right)\cancel{-4} =$

$= x$

$(g \circ f)(x) =$

$\frac{1}{\cancel{3}}(\cancel{3}x \cancel{-4} \overset{0}{\cancel{+4}}) =$

✓ $= x$

Finding the components of a composite function

Look for a function that is inside the formula, expressions that are in parentheses, under radical signs or in the denominator. Very important concept for differential Calculus!!

Find the functions f and g such that $f \circ g = H$, if $H(x) = (x^2 + 1)^{50}$

$f(g(x)) = (x^2+1)^{50}$

Inside function =

$g(x) = x^2+1$

Outside function =

$\bigcirc^{50}$

$f(x) = x^{50}$

Find the functions f and g when

$f \circ g = H$, if $H(x) = \dfrac{1}{x+1}$

$f(g(x)) = \dfrac{1}{x+1}$

inside function =

$g(x) = x+1$

Outside function =

$\dfrac{1}{\bigcirc}$ (Fraction)

$f(x) = \dfrac{1}{x}$

Precalculus

Lesson 5.2: One to One Functions; Inverse Functions

Mrs. Snow, Instructor

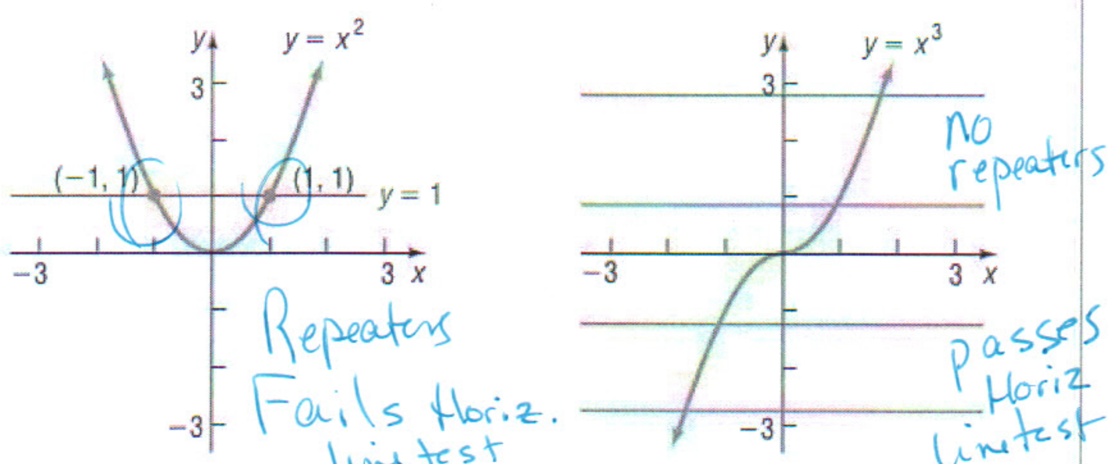
A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range. That is, if x_1 and x_2 are two different inputs of a function f, then f is one-to-one if $f(x_1) \neq f(x_2)$.

Translation. A function is one-to-one if every output is unique, there are no repeating outputs

Graphically we can determine a one-to-one relationship by using the **horizontal-line-test** to determine of *f* is one-to-one. Basically, this is analogous to the vertical line test, only horizontal.

Horizontal-line Test

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.



(a) A horizontal line intersects the graph twice; *f* is not one-to-one

(b) Every horizontal line intersects the graph exactly once; *g* is one-to-one

Inverse function

If a function is one-to-one with a domain **D** and a range **R**, then the inverse function of f, is f^{-1} the inverse function will have a domain **R** and a range **D**

Domain of f = Range of f^{-1} Range of f = Domain of f^{-1}

and

$$f(g(x)) = g(f(x)) = x$$

Inverses: An inverse function is a function that undoes the action of another function. Another way of saying inverse is opposite. Did you ever play "opposite day" with your parents? If you remember you probably would not confess it; Yes means No and No means Yes! Mathematically: x *is* y *and* y *is* x.

Find the inverse of the following one-to-one function: ← Switch x & y

f(x): $\{(-3, -27), (-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)\}$

D: $\{-3, -2, -1, 0, 1, 2, 3\}$ $f^{-1}(x)$ D $\{-27, -8, -1, 0, 1, 2, 8, 27\}$

R: $\{-27, -8, -1, 0, 1, 8, 27\}$ R: $\{-3, 2, -1, 0, 1, 2, 3\}$

$f^{-}(x) = \{(-27, -3), (-8, -2), (-1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$

(a) Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$. Composite Must = X

$g(g^{-1}(x)) = (\sqrt[3]{x})^3 = x$

$g^{-}(g(x)) = \sqrt[3]{x^3} = x$

Yes, Inverses

(b) Verify that the inverse of $f(x) = 2x + 3$ is $f^{-1}(x) = \frac{1}{2}(x - 3)$

$f(f^{-1}(x)) = \cancel{2}(\tfrac{1}{\cancel{2}}(x - \cancel{3})) + \cancel{3} = x$

$f^{-}(f(x)) = \frac{1}{\cancel{2}}((\cancel{2}x + \cancel{3}) - \cancel{3}) = X$ Yes, Inverses

How to Find the Inverse Function

Find the inverse of $f(x) = 2x + 3$.

Step 1 Replace $f(x)$ with y. In $y = f(x)$, interchange the variables x and y to obtain $x = f(y)$. This equation defines the inverse function f^{-1} implicitly.

$y = 2x + 3$ — x is y and y is x

$x = 2y + 3$

Step 2 If possible, solve the implicit equation for y in terms of x to obtain the explicit form of f^{-1}, $y = f^{-1}(x)$.

$\frac{1}{2}(2y) = (x - 3)\frac{1}{2}$

$y = \frac{1}{2}(x-3) = f^{-1}$

Step 3 Check the result by showing that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

Check

$\cancel{2}\left(\cancel{\tfrac{1}{2}}(x\cancel{-3})\cancel{+3}\right) = x$ ✓

$\frac{1}{2}(2x\cancel{+3-3}) = x$ ✓

The following function is one-to-one. Find its inverse and check the result.

$$f(x) = \frac{2x+1}{x-1}, x \neq 1$$

$y = \frac{2x+1}{x-1}$ Switch

$x = \frac{2y+1}{y-1}$

$x(y-1) = 2y+1$

$xy - x = 2y + 1$

$xy - x = 2y + 1$

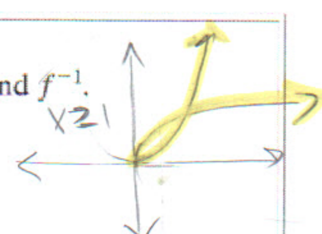
$xy - 2y = x + 1$

$y(x-2) = x+1$

$y = \frac{x+1}{x-2} = f^{-1}$

If a function is not one-to-one, it has no inverse function. However, by restricting the domain of that function, we can make the function 1-1 and find its inverse.

Find the inverse of $y = f(x) = x^2$ if $x \geq 0$. Graph f and f^{-1}.



$y = x^2 \xrightarrow[f^{-1}]{} \sqrt{x} = \sqrt{y^2}$

$y = \pm\sqrt{x}$ Not a function!

*Restrict $f(x)$ domain so f^{-1} range then has a restriction

$y = x^2 \longrightarrow \boxed{y = \sqrt{x} \quad R: y \geq 0}$

D: $x \geq 0$

Remember $f(x)$ Domain $= f^{-1}$ Range

$f(x)$ Range $= f^{-1}$ Domain