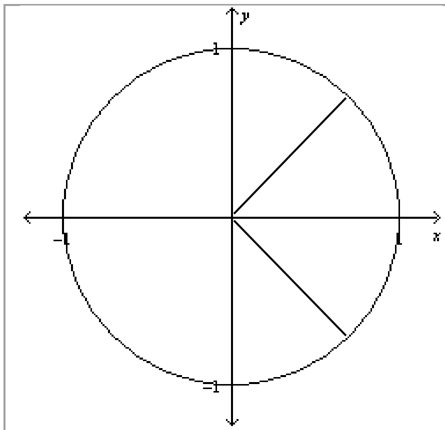
Precalculus
Lesson 6.5: Graphs of the Tangent, Cotangent Cosecant, and Secant Functions
Mrs. Snow, Instructor

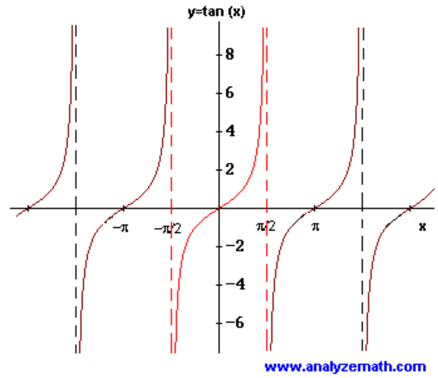
Tangent function facts:

- $y = \tan \omega x \qquad period = \frac{\pi}{\omega}$
- $\tan x = \frac{\sin x}{\cos x}, \quad \therefore \; when \; \sin x = 0, \quad \tan x = 0$
  $when \; \cos x = 0, \quad \tan x \; is \; undefined!$
- Tangent graph will have asymptotes at values of x where the function is undefined: $x = \frac{\pi}{2} and -\frac{\pi}{2}$.



Period: $\boldsymbol{\pi}$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

input $\boldsymbol{x}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\boldsymbol{y = tanx}$					

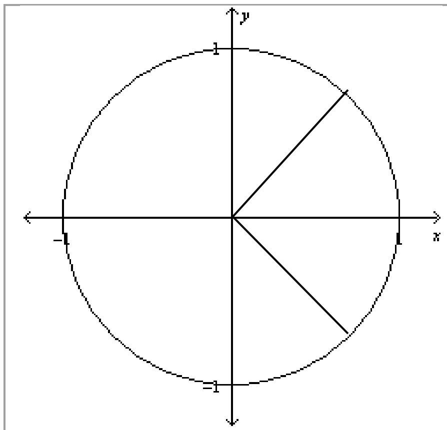


Domain: all real numbers except odd mult. of $\frac{\pi}{2}$

Range: $(-\infty, \infty)$

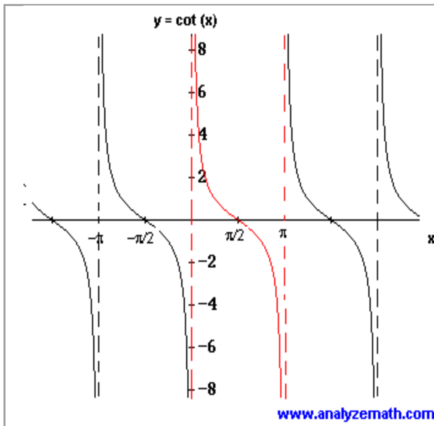
Cotangent function facts:

- $y = \cot \omega x \qquad period = \frac{\pi}{\omega}$
- $\cot x = \frac{\cos x}{\sin x} \quad \therefore \, when \cos x = 0, \quad \cot x = 0$
- $\qquad\qquad when \sin x = 0, \quad \cot x \; is \; undefined$
- Asymptotes are found at π and multiples of π.



Period: $\boldsymbol{\pi}$ $\qquad [\mathbf{0}, \boldsymbol{\pi}]$

input $\boldsymbol{x}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\boldsymbol{y = cot\theta}$					



Domain: all real numbers except mult. of π

Range: $(-\infty, \infty)$

The same process used for sine and cosine may be followed for these trig functions. First let's look at the equations:

$$y = A\ \tan(\omega x) + B \qquad period = \frac{\pi}{\omega} \qquad y = A\cot(\omega x) + B$$

For tangent an appropriate interval is $\left(-\frac{\pi}{2\omega}, \frac{\pi}{2\omega}\right)$

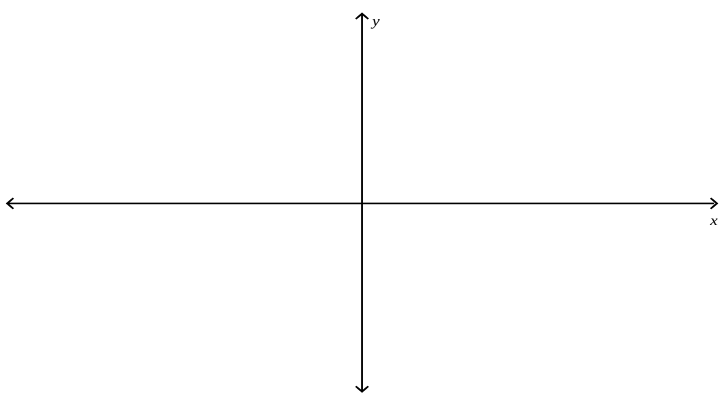
for cotangent and appropriate interval is: $\left(0, \frac{\pi}{\omega}\right)$

The intervals are bounded by vertical asymptotes.

$$y = 2\tan x - 1$$

$$T = \qquad\qquad A =$$

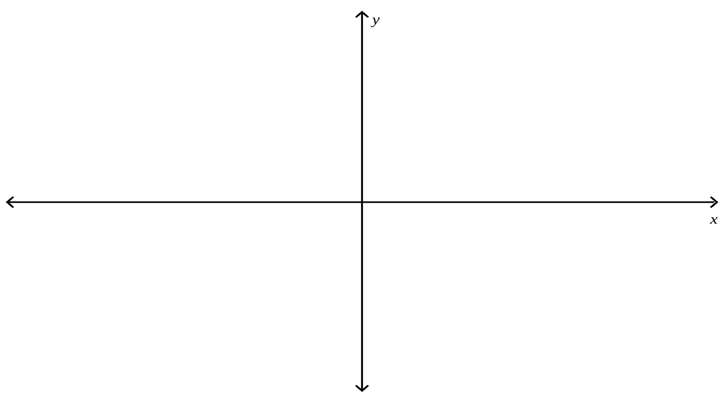
x					
$\tan x$					
$2\tan x$					
$2\tan x$ -1					



$$y = 3\tan(2x)$$

$period =$ $A =$

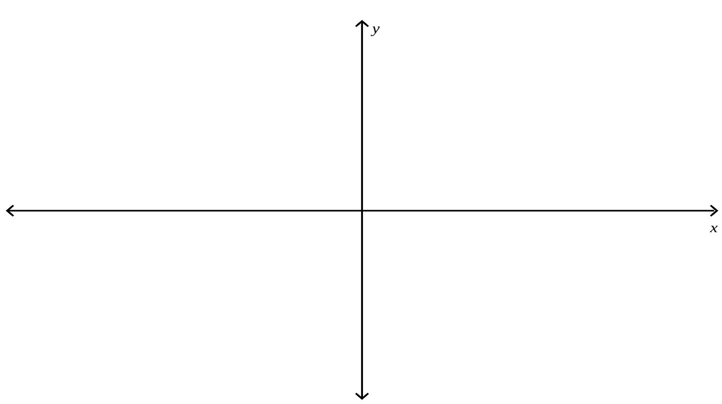
x					
$2x$					
$\tan 2x$					
$3\tan 2x$					



$\boldsymbol{y = A\cot(\omega x) + B}$ $\boldsymbol{period = \frac{\pi}{\omega}}$ **For cotangent and appropriate interval is: $\boldsymbol{\left(0, \frac{\pi}{\omega}\right)}$.**

$$y = \cot x + 2$$

x					
$\cot x$					
$\cot x + 2$					



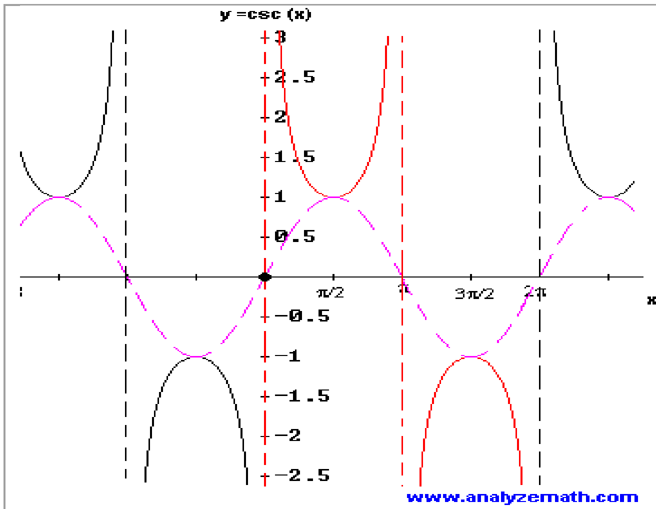
Cosecant and Secant Functions

The cosecant and secant functions are **reciprocal functions.** These functions are graphed by first graphing sine or cosine. Where the sine and cosine functions have zeros the reciprocals of cosecant secant functions will be undefined, hence, asymptotes.

To graph csc θ First graph $sin\ \theta$

Domain: all real numbers except multiples of π
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

input x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{2\pi}{3}$	$\frac{3\pi}{2}$	2π
$y = sin\ x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$\frac{\sqrt{3}}{2}$	-1	0
$y = csc\ x = 1/(sin\ x)$	U	2	$\frac{2\sqrt{3}}{3}$	1	U	$\frac{2\sqrt{3}}{3}$	-1	U



Secant is like cosecant in that it is reciprocal function. So plot the cosine function, locate vertical asymptotes at $\cos x = 0$, and graph the secant function.

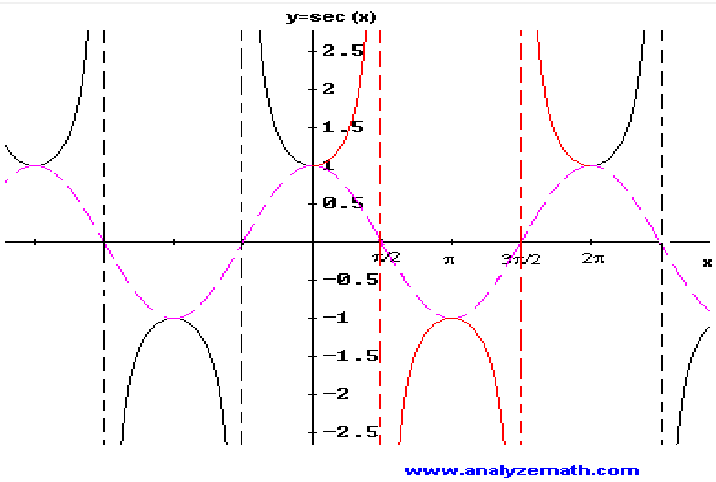
Graph $sec\theta$. First plot $\cos x$.

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

input x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
$y = cos\, x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	-1	0	1
$y = sec\, x$	1	$\frac{2\sqrt{3}}{3}$	2	U	2	-1	U	1



note: where the sine and cosine functions are equal to zero, this is where we see the asymptotes for the reciprocals cosecant and secant. Understand that this will remain the same even when we have a vertical slide.

$y = A\csc \omega x + B$ $y = A\sec \omega x + B$ $period = \frac{2\pi}{\omega}$

1. graph the reciprocal function of sine or cosine, *the guide functions,*
2. Dash in the guide function.
3. Sketch vertical asymptotes; these occur at the x-values for which the guide function equals 0
4. now add the vertical slide up or down of **B** units
5. Sketch the typical U-shaped branches, approaching the asymptotes that typify the cosecant and secant functions. Note the function's minimum is its reciprocal's maximum.

$$y = 2\csc x - 1$$

$T =$ $A =$

$$\textbf{first graph "guide function": } y = 2\sin x - 1$$