Precalculus

Lesson 5.2: One to One Functions; Inverse Functions

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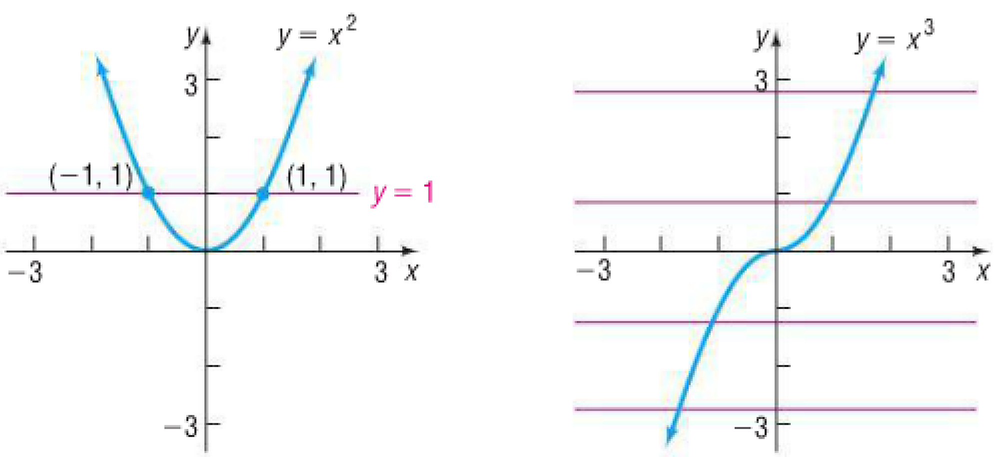
A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range. That is, if x_1 and x_2 are two different inputs of a function f, then f is one-to-one if $f(x_1) \neq f(x_2)$.

Translation. A function is one-to-one if every output is unique, there are no repeating outputs

Graphically we can determine a one-to-one relationship by using the **horizontal-line-test** to determine of *f* is one-to-one. Basically, this is analogous to the vertical line test, only horizontal.

Horizontal-line Test

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.



(a) A horizontal line intersects the graph twice; *f* is not one-to-one

(b) Every horizontal line intersects the graph exactly once; *g* is one-to-one

$f(x)$ *and* $g(x)$ are inverses if and only if:

$$f(g(x)) = g(f(x)) = x$$

Suppose that f is a one-to-one function. Then, to each x in the domain of f, there is exactly one y in the range (because f is a function); and to each y in the range of f, there is exactly one x in the domain (because f is one-to-one). The correspondence from the range of f back to the domain of f is called the **inverse function of *f*.** The symbol f^{-1} is used to denote the inverse of f.

Domain of f = Range of f^{-1} Range of f = Domain of f^{-1}

Inverses: Another way of saying inverse is opposite. Did you ever play "opposite day" with your parents? If you remember you probably would not confess it; Yes means No and No means Yes! Mathematically: $x\ is\ y\ and\ y\ is\ x$.

Find the inverse of the following one-to-one function:

$$\{(-3, -27), (-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)\}$$

(a) Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$.

(b) Verify that the inverse of $f(x) = 2x + 3$ is $f^{-1}(x) = \frac{1}{2}(x - 3)$

How to Find the Inverse Function

Find the inverse of $f(x) = 2x + 3$. Graph f and f^{-1} on the same coordinate axes

Step 1 Replace $f(x)$ with y. In $y = f(x)$, interchange the variables x and y to obtain $x = f(y)$. This equation defines the inverse function f^{-1} implicitly.

Step 2 If possible, solve the implicit equation for y in terms of x to obtain the explicit form of f^{-1}, $y = f^{-1}(x)$.

Step 3 Check the result by showing that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

The following function is one-to-one. Find its inverse and check the result.

$$f(x) = \frac{2x+1}{x-1}, x \neq 1$$

If a function is not one-to-one, it has no inverse function. However, by restricting the domain of that function, we can make the function 1-1 and find its inverse.

Find the inverse of $y = f(x) = x^2$ if $x \geq 0$. Graph f and f^{-1}.