

Precalculus
Lesson 4.6: Polynomial and Rational Inequalities
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This section covers the processes to graph inequalities of polynomials and rational functions

Solution

1. Write the inequality so that a polynomial/rational expression is on the left side and 0 is on the right side
2. Determine the real zeros (x-intercepts) of f and any real numbers for which the expression is undefined.
3. Using the zeros and undefined values, divide the real number line into intervals
 - a. Is the inequality $<$, $>$, \leq , or \geq at zero?
 - b. Equality means a point on the zero
 - c. Not equal means a circle
4. Select a number in each interval, evaluate at that number. Focus on the sign of the factors and the overall outcome of \pm . Don't worry about the exact numerical answer.

Solve the inequalities algebraically and graph the solution

$$x^4 > x$$

$$\frac{4x + 5}{x + 2} \geq 3$$

textbook pg. 240

SUMMARY Steps for Solving Polynomial and Rational Inequalities Algebraically

STEP 1: Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \quad f(x) \geq 0 \quad f(x) < 0 \quad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient and find the domain of f .

STEP 2: Determine the real numbers at which the expression f equals zero and, if the expression is rational, the real numbers at which the expression f is undefined.

STEP 3: Use the numbers found in Step 2 to separate the real number line into intervals.

STEP 4: Select a number in each interval and evaluate f at the number.

(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.

(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.

If the inequality is not strict (\geq or \leq), include the solutions of $f(x) = 0$ that are in the domain of f in the solution set. Be careful to exclude values of x where f is undefined.