

Precalculus
Lesson 12.1: Sequences
Mrs. Snow, Instructor

A **sequence** is a function f whose domain is the set of positive integers. The values $f(1), f(2), f(3), \dots$ are called terms.

For Example:

$$-2, 4, 6, 8, \dots, 2n$$

Since we are talking about a function, we can graph sequences.

Write down the first six terms of the following sequence.

$$\{a_n\} = \left\{ \frac{n-1}{n} \right\}$$

Write down the first six terms of the following sequence.

$$\{b_n\} = \left\{ (-1)^{n+1} \left(\frac{2}{n} \right) \right\}$$

Write down the first six terms of the following sequence.

$$\{c_n\} = \begin{cases} n & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd} \end{cases}$$

Determining a Sequence from a Pattern

Number the terms and see what happens between each term:

(a) $e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \dots$

(b) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

(c) $1, 3, 5, 7, \dots$

(d) $1, 4, 9, 16, 25, \dots$

(e) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \dots$

Factorial

A factorial is a non negative integer written with an exclamation mark. If $n \geq 0$ is an integer, the factorial is defined as follows:

$$0! = 1 \quad \text{and} \quad 1! = 1$$

$$n! = n(n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1 \quad \text{for } n \geq 2$$

and.....

$$n! = n(n-1)!$$

Solve:

$$9!$$

$$\frac{12!}{10!}$$

$$\frac{3! 7!}{4!}$$

A Sequence Defined by a Recursive Formula

A second way of defining a sequence is to assign a value to the first term and specify the n th term by a formula or equation that involves one or more of the terms preceding it. The sequence is defined **recursively**, and the formula is a **recursive formula**.

Write the first 5 terms of the recursive sequence

$$u_1 = 1, u_2 = 1, u_n = u_{n-2} + u_{n-1}$$

Sigma Notation: a short cut notation to indicate the sum of some or all of the terms of a sequence.

Given a sequence

$$a_1, a_2, a_3, a_4, \dots a_n.$$

we can write the sum of the first n terms using **summation notation**, or **sigma notation**.

The notation derives its name from the Greek Letter Σ . This corresponds to our S for "sum."

The following notation is used

$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots a_n$$

k is called the index of summation, it is basically the starting number for the sequence.

Write out each sum

$$\sum_{k=1}^{10} \frac{1}{k}$$

$$\sum_{k=1}^n k!$$

Express each sum using summation notation

$$1^2 + 2^2 + 3^2 + \dots + 9^2$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{n-1}}$$

The following sums are natural consequences of properties of the real numbers. These are useful for adding terms of a sequence algebraically:

Properties of Sequences

If $\{a_n\}$ and $\{b_n\}$ are two sequences and c is a real number, then:

$$\sum_{k=1}^n (ca_k) = ca_1 + ca_2 + \cdots + ca_n = c(a_1 + a_2 + \cdots + a_n) = c \sum_{k=1}^n a_k$$

$$\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

$$\sum_{k=1}^n (a_k - b_k) = \sum_{k=1}^n a_k - \sum_{k=1}^n b_k$$

$$\sum_{k=j+1}^n a_k = \sum_{k=1}^n a_k - \sum_{k=1}^j a_k, \quad \text{where } 0 < j < n$$

Find the sums:

$$\sum_{k=1}^5 k^2$$

$$\sum_{j=3}^5 \frac{1}{j}$$

$$\sum_{i=5}^{10} i$$

$$\sum_{i=1}^6 2$$