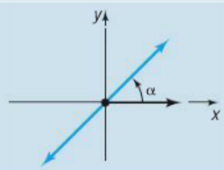
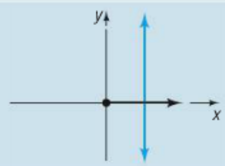
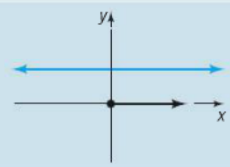
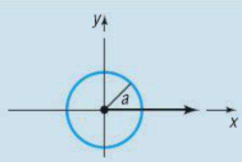
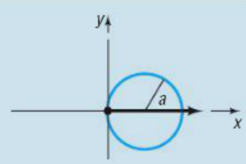
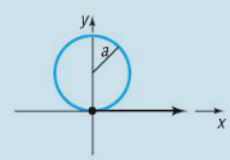
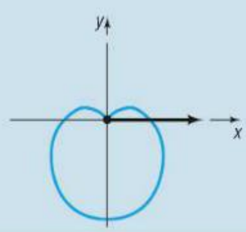
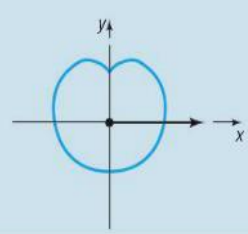
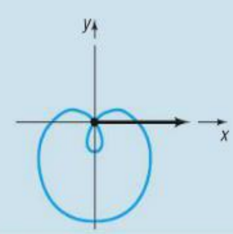
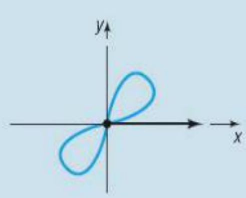
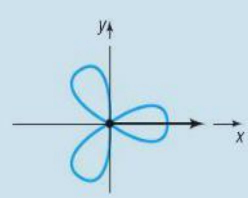
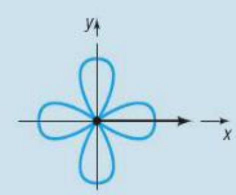
Precalculus
Lesson 9.2 Graphs of Polar Equations
Mrs. Snow, Instructor

As we studied last section points may be described in polar form or rectangular form. Likewise an equation may be written using either polar or rectangular coordinates. Depending on specific equation, one form may be easier to understand and graph than the other. Below are some common polar graphs and their equations written in both polar and rectangular forms.

Lines			
Description	Line passing through the pole making an angle α with the polar axis	Vertical line	Horizontal line
Rectangular equation	$y = (\tan\alpha)x$	$x = a$	$y = b$
Polar equation	$\theta = \alpha$	$r\cos\theta = a$	$r\sin\theta = b$
Typical graph			

Circles			
Description	Center at the pole, radius a	Passing through the pole, tangent to the line $\theta = \frac{\pi}{2}$, center on the polar axis, radius a	Passing through the pole, tangent to the polar axis, center on the line $\theta = \frac{\pi}{2}$, radius a
Rectangular equation	$x^2 + y^2 = a^2, \quad a > 0$	$x^2 + y^2 = \pm 2ax, \quad a > 0$	$x^2 + y^2 = \pm 2ay, \quad a > 0$
Polar equation	$r = a, \quad a > 0$	$r = \pm 2a\cos\theta, \quad a > 0$	$r = \pm 2a\sin\theta, \quad a > 0$
Typical graph			

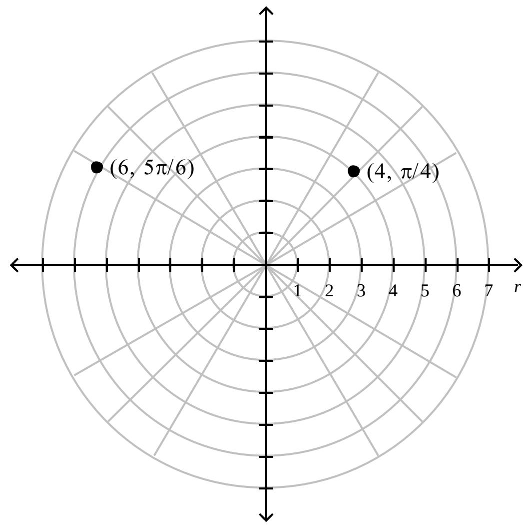
Other Equations			
Name	Cardioid	Limaçon without inner loop	Limaçon with inner loop
Polar equations	$r = a \pm a\cos\theta, \quad a > 0$	$r = a \pm b\cos\theta, \quad 0 < b < a$	$r = a \pm b\cos\theta, \quad 0 < a < b$
	$r = a \pm a\sin\theta, \quad a > 0$	$r = a \pm b\sin\theta, \quad 0 < b < a$	$r = a \pm b\sin\theta, \quad 0 < a < b$
Typical graph			
Name	Lemniscate	Rose with three petals	Rose with four petals
Polar equations	$r^2 = a^2\cos(2\theta), \quad a > 0$	$r = a\sin(3\theta), \quad a > 0$	$r = a\sin(2\theta), \quad a > 0$
	$r^2 = a^2\sin(2\theta), \quad a > 0$	$r = a\cos(3\theta), \quad a > 0$	$r = a\cos(2\theta), \quad a > 0$
Typical graph			

To plot points with polar coordinates, it is convenient to use a polar grid. It is sort of like the unit circle superimposed with graph paper, like below:

Special graphs:
θ = constant – graphs a line at angle θ
r=constant – graphs a circle of radius r

Sketch the graph of the equation and express the equation in rectangular coordinates:

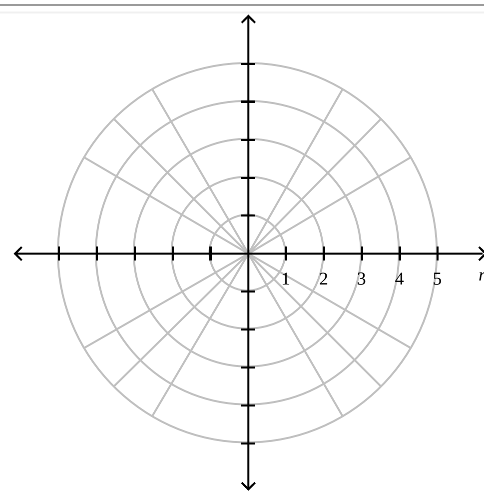
$r = 3$

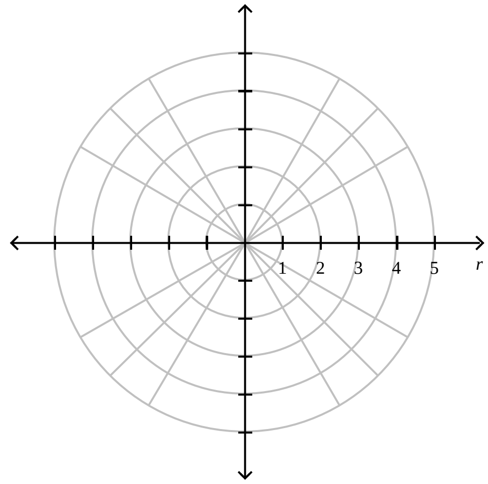
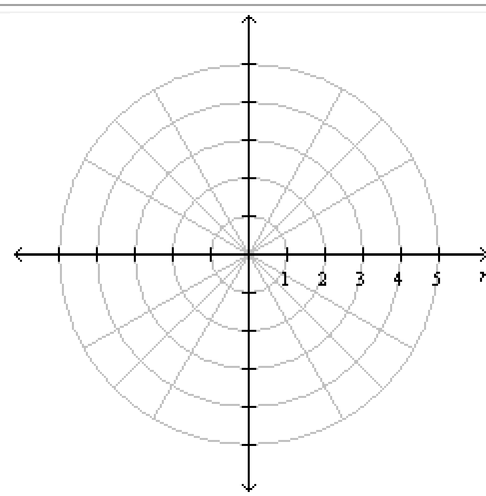


Graphing a Polar Equation of a Line:
Some equations can easily be expressed in rectangular coordinates. If this is the case then convert to rectangular coordinates.

Identify and graph the equation

$$\theta = \frac{\pi}{4}$$

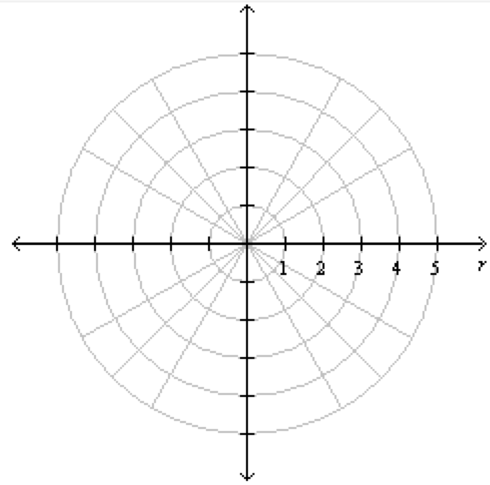


Identify and graph the equation $$r\sin\theta = 2$$	
Identify and graph the equation $$r\cos\theta = -3$$	

Graphing a Circle

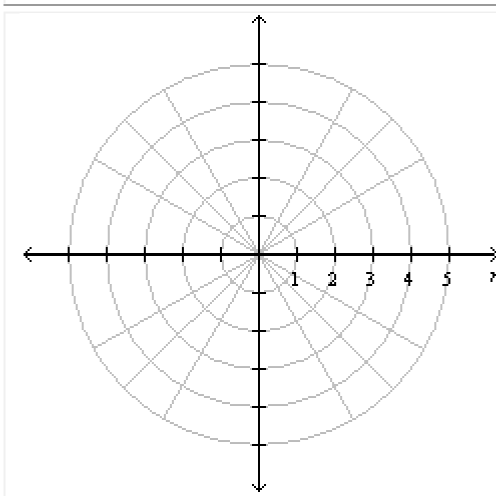
Sketch the polar equation (transform the equation into its rectangular form)

$$r = 4 \sin \theta$$



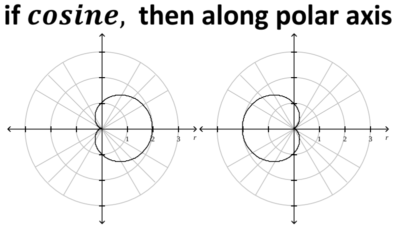
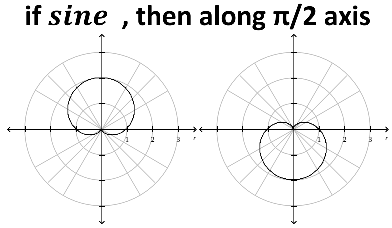
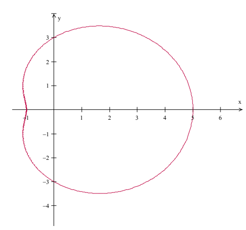
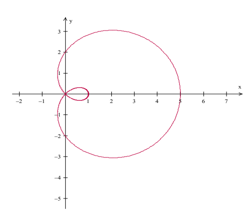
Sketch the polar equation

$$r = -2 \cos \theta$$



Other Equations (pg. 581)

Name	Cardioid	Limaçon **no** inner loop has a dimple	Limaçon inner loop
Polar Equation	$r = a \pm \mathrm{a}\cos\theta$ $r = a \pm \mathrm{a}\sin\theta$ $a > 0$	$r = a \pm \mathrm{b}\cos\theta$ $r = a \pm \mathrm{b}\sin\theta$ $0 < b < a$	$r = a \pm \mathrm{b}\cos\theta$ $r = a \pm \mathrm{b}\sin\theta$ $0 < a < b$

Cardioid graphs	**Limaçon graphs**
<u>$a > 0,$ $\quad$ *distance on axis is* $2a$</u> **if *cosine*, then along polar axis**  $r = 1 + \cos\theta$ $\qquad$ $r = 1 - \cos\theta$ **if *sine* , then along π/2 axis**  $r = 1 + \sin\theta$ $\qquad$ $r = 1 - \sin\theta$	**if *cosine*: along polar axis** **if *sine*: along $\frac{\pi}{2}$ axis** **a.** Limacon no inner loop if : $a > b$  **b.** Limacon has an inner loop if: $a < b$ 

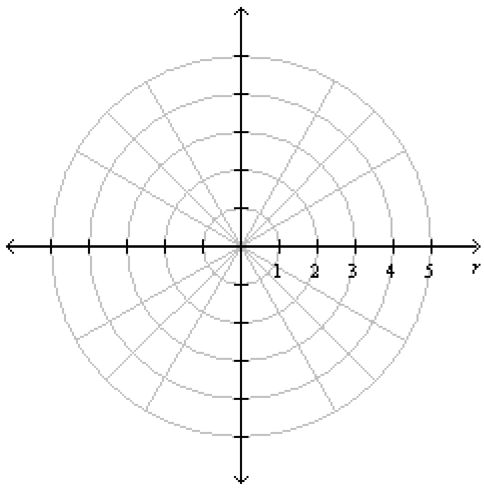
Cardioid – heart shaped (pg. 581)

graph $r = 2 - 2\sin\theta$

numbers indicate shape_______________ equation has sine so along

__________________axis

Negative means: length =_________



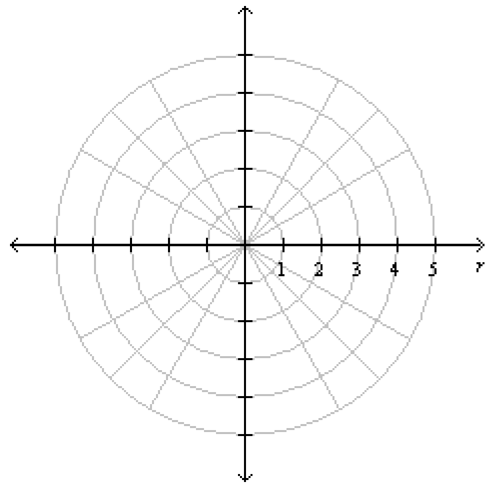
Whenever you cannot remember how to graph the polar equation, you can always graph a period of the trig function from $0 \leq \theta < 2\pi$ and transfer the data over to a polar graph. Don't rely on memorizing an equation and associated graph shape, you will want a backup method!!

Table of values *(use values for theta that yield friendly values for r):*

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	1/2	1	1/2	0	-1	0
$r = 2 - 2\sin\theta$	2	1	0	1	2	4	2

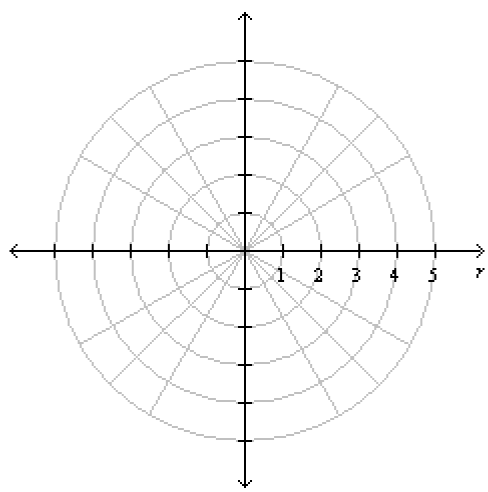
Graphing a limaçon without an inner loop

Sketch the graph of the equation
$r = 3 + 2cos\,\theta$



Graphing a limaçon with an inner loop

$$r = 1 + 2\,cos\,\theta$$

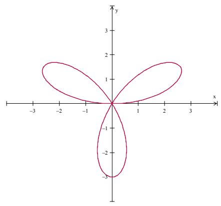


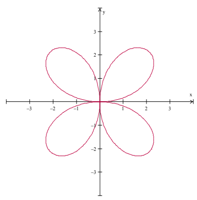
More Equations

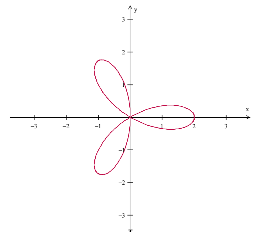
Rose with even/odd petals

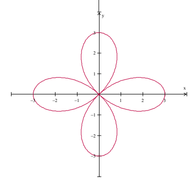
$$n: \begin{cases} odd = n \ petals \\ even = 2n \ petals \end{cases} \qquad a = length\ of\ petal$$

$$\boldsymbol{r = a \sin n\theta} \qquad \boldsymbol{r = a \cos n\theta}$$



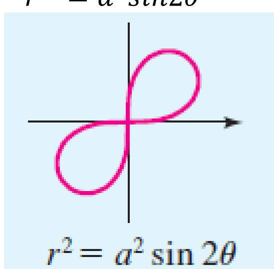




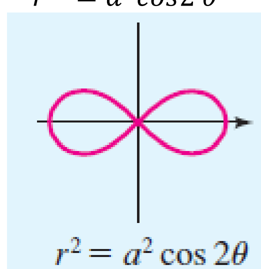


Lemniscates – Figure 8 shaped curves

$r^2 = a^2 sin2\theta$

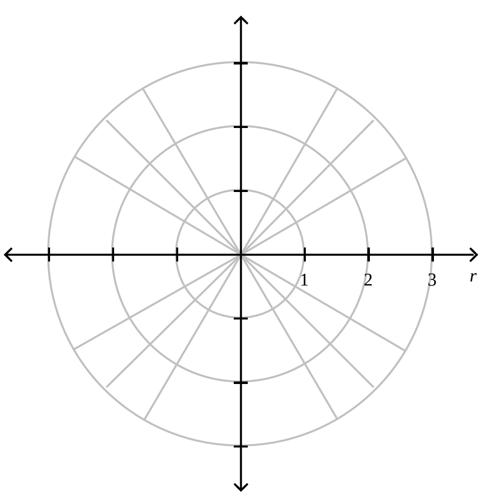


$r^2 = a^2 cos2\,\theta$ $\quad a = petal\ length$

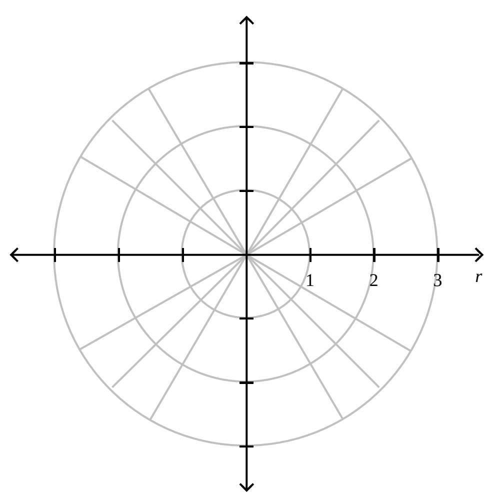


Graphing a Polar Equation: n-leaved rose (petals)

$r = 2\sin 3\theta$

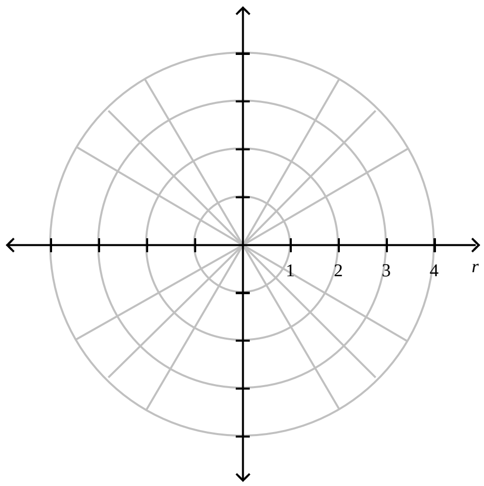


$r = 2\cos 2\theta$

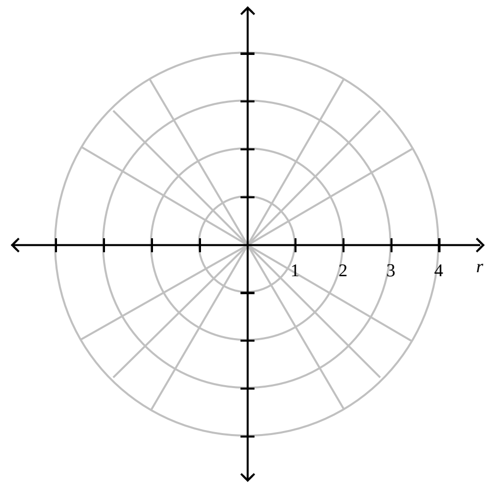


Lemniscates – Figure 8 shaped curves

$r^{2} = 9sin2\theta$



$r^{2} = 2^{2}cos2\theta$



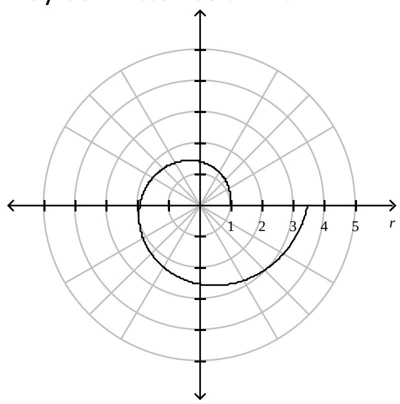
Graphing a Polar Equation (spiral)

It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

There are several equations that will produce a spiral. The **logarithmic spiral**

$$r = e^{\theta/5}$$

may be written as $\theta = 5\ln r$



Archimedes Spiral is in the form of

$$r = a\theta$$