Precalculus
Lesson 7.1: The Inverse Sine, Cosine and Tangent Functions
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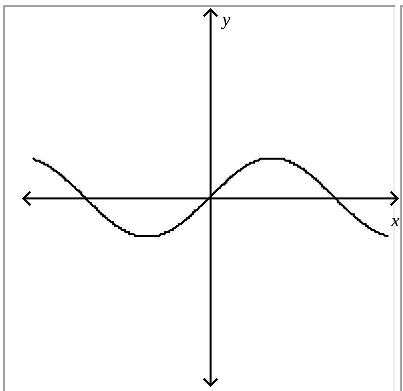
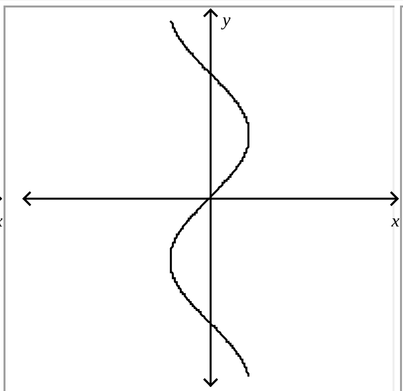
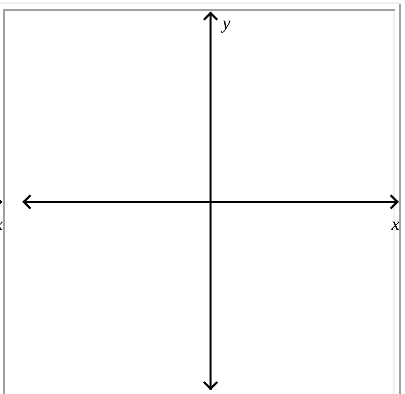
Inverse: A mathematical operation that is the opposite effect of another operation. The operation undoes what the first operation did! Some examples of inverse operations include addition and subtraction and multiplication and division.

In 5.2 we studied inverse functions. If a function is one-to-one if has an inverse (one function undoes the other). We are able to restrict the domain of a function so to make it one-to-one. While trig functions are of course functions, they are not all 1 – 1, so they may not have inverses. We can, however, force our trig functions into being 1 – 1 by limiting their domain.

Inverse Sine Function: $\boldsymbol{sin^{-1}}$ is *also known as* *<u>arcsine</u>* *and written as* *<u>arcsin</u>*

	 not a function	
$\sin x = y$ Domain: Range:	$\sin^{-1} x = y$	$\sin^{-1} x = y$ Domain: Range:

Definition:

$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$
$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$What????$$
$$y = sin^{-1} x$$
$$\sin y = \sin sin^{-1} x$$
$$\therefore x = \sin y$$

Remember, that inverses undo one another, so if I take the inverse sine of sine, the operation is undone!

Finding the exact value of an inverse sine function:

$sin^{-1} 1$	$sin^{-1} -\frac{1}{2}$	$sin^{-1} \frac{3}{2}$

Approximate values of inverse sine functions may be found using a calculator. Express the answer in radians rounded to 2 decimal places.

$sin^{-1}\frac{1}{3}$	$sin^{-1}\left(-\frac{1}{4}\right)$

For composite functions, the domain is dependent upon the domain restrictions of the inner function. This is crucial when determining the value of a composite function where the inner function is outside the domain restrictions.

In the terms of the sine function and its inverse, we have the following properties:

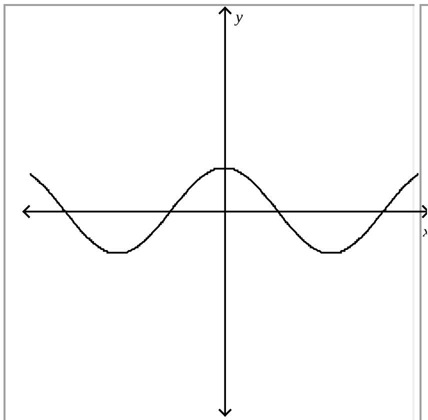
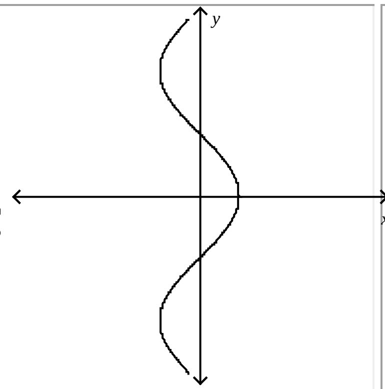
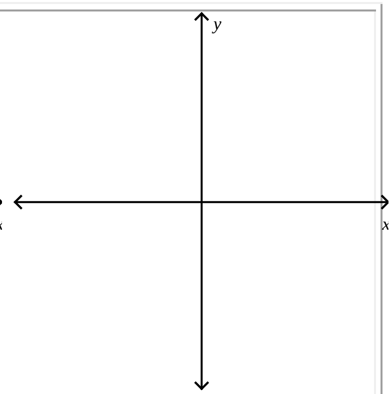
$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

Find the exact value of composite functions:

$sin^{-1}(\sin\frac{\pi}{8})$	$sin^{-1}(\sin\frac{5\pi}{8})$

Inverse Cosine Function: $\boldsymbol{cos^{-1}}$ *also called* <u>*arrcosine*</u> *and written as* <u>*arccsos*</u>

		
$cos\ x = y$ Domain: Range:	$\cos^{-1} x = y$	$\cos^{-1} x = y$ Domain: Range:

$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y$$
$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad 0 \le y \le \pi$$

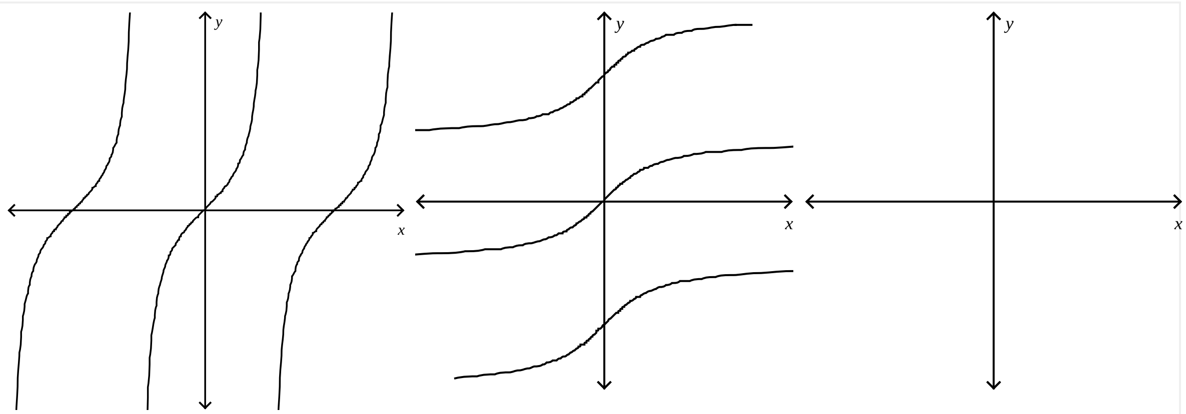
$\cos^{-1} 0$	$\cos^{-1} -\frac{\sqrt{2}}{2}$

In the terms of the cosine function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \qquad \text{where } 0 \le x \le \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

$cos^{-1}\left(\cos\left(\frac{\pi}{12}\right)\right)$	$cos^{-1}[\cos\left(-\frac{2\pi}{3}\right)]$
$\cos(cos^{-1}\pi)$	$\cos\left(cos^{-1}(-0.4)\right)$

Inverse Tangent Function: $\boldsymbol{tan^{-1}}$*also called* *<u>arctangent</u>* *and written as* *<u>arctan</u>*



$tan\ x = y$

Domain:

Range:

$\tan^{-1} x = y$

Domain:

Range:

The inverse tangent function is the function tan^{-1} with domain of all real numbers and range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$defined by

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$
$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

EXAMPLE Evaluate the inverse tangent functions

$\tan^{-1} 1$	$\tan^{-1} -\sqrt{3}$
$y = \tan^{-1}(-20)$	

In the terms of the tangent function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where } -\infty < x < \infty$$