Precalculus
Lesson 5.1: Composite Functions
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Composite Functions: A composite function is a function that is made or composed of more than one "independent" function. In general, a number *x* is applied to one function the result or output is then applied to a second function.

Given two functions f and g, the **composite function,** denoted by $f \circ g$ (read as "f composed with g"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Domain of a composite:
The domain of a composite function, $f \ o \ g$, if defined whenever both $g(x) and \ f\big(g(x)\big)$ are defined.

Evaluating a composite function

Suppose that $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find:

(a) $(f \circ g)(1)$ (b) $(g \circ f)(1)$ (c) $(f \circ f)(-2)$ (d) $(g \circ g)(-1)$

Finding a composite function and its domain

Suppose that $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$.

Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.

Finding the domain of a composite function

1. $g(x)$ must be defined so any x not in the domain of g must be excluded.
2. $f(g(x))$ must be defined so any x for which $g(x)$ is not in the domain of f mus be excluded.

Suppose that $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{4}{x-1}$.

Find the domain of $(f \ o \ g)(x)$

Show that two composite functions are equal

If $f(x) = 3x - 4$ and $g(x) = \frac{1}{3}(x + 4)$, show that

$$(f \circ g)(x) = (g \circ f)(x) = x$$

for every x in the domain of $f \circ g$ and $g \circ f$.

Finding the components of a composite function

Find functions f and g such that $f \circ g = H$ if $H(x) = (x^2 + 1)^{50}$.

Find functions f and g such that $f \circ g = H$ if $H(x) = \dfrac{1}{x+1}$.