Precalculus

Lesson4.5: The Graph of a Rational Function

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Calculators of course make graphing rational function much easier and quicker. However, we need to be proficient in using algebraic analysis to draw conclusions of the graph.

textbook pg. 228

SUMMARY

Analyzing the Graph of a Rational Function

STEP 1: Factor the numerator and denominator of R. Find the domain of the rational function.

STEP 2: Write R in lowest terms.

STEP 3: Locate the intercepts of the graph. The x-intercepts, if any, of $R(x) = \dfrac{p(x)}{q(x)}$ in lowest terms satisfy the equation $p(x) = 0$. The y-intercept, if there is one, is $R(0)$.

STEP 4: Locate the vertical asymptotes. The vertical asymptotes, if any, of $R(x) = \dfrac{p(x)}{q(x)}$ in lowest terms are found by identifying the real zeros of $q(x)$. Each zero of the denominator gives rise to a vertical asymptote.

STEP 5: Locate the horizontal or oblique asymptote, if one exists, using the procedure given in Section 4.4. Determine points, if any, at which the graph of R intersects this asymptote.

STEP 6: Graph R using a graphing utility.

STEP 7: Use the results obtained in Steps 1 through 6 to graph R by hand.

Analyze the rational function:

$$R(x) = \frac{x-1}{x^2-4}$$

Analyze the rational function:

$$\frac{x^2 - 1}{x}$$

Analyze the rational function:

$$\frac{x^4 + 1}{x^2}$$

Analyze the rational function:

$$\frac{3x^2 - 3x}{x^2 + x - 12}$$

Analyze the rational function with a Hole

$$\frac{2x^2 - 5x + 2}{x^2 - 4}$$

Finding the Least Cost

Reynolds Metal Company manufactures aluminum cans in the shape of a cylinder with a capacity of 500 cubic centimeters (cm^3), or $\frac{1}{2}$ liter. The top and bottom of the can are made of a special aluminum alloy that costs $0.05¢$/per square centimeter (cm^2). The sides of the can are made of material that costs $0.02¢/\text{cm}^2$.

(a) Express the cost of material for the can as a function of the radius r of the can.
(b) Use a graphing utility to graph the function $C = C(r)$.
(c) What value of r will result in the least cost?
(d) What is this least cost?