Precalculus
Lesson 4.6: Polynomial and Rational Inequalities
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This section covers the processes to graph inequalities of polynomials and rational functions

Solution

1. Write the inequality so that a polynomial/rational expression is on the left side and 0 is on the right side
2. Determine the real zeros (x-intercepts) of *f*. Rational: real numbers for which the expression is undefined.
3. Using the zeros divide the real number line into intervals
   a. Is the inequality $<, >, \leq, or \geq$ at zero?
   b. Equality means a point on the zero
   c. No equal means a circle
4. Select a number in each interval, evaluate at that number. Focus on the sign of the factors and the overall outcome of $\pm$. Don't worry about the exact numerical answer.

Solve the inequalities algebraically and graph the solution

$x^4 > x$

① $x^4 - x > 0$

$x(x^3-1) = 0$ ← Sum
Know difference of two cubes formula

$x(x-1)(x^2+x+1) = 0$

② $x = 0$ $\quad x - 1 = 0$ $\quad x^2+x+1=0$

$x = 1$ $\quad$ No real solutions (not factorable)

Intervals > 0

③ 

test points

Factors			
x	−	+	+
$x-1$	−	−	+
x^2+x+1	+	+	+
	+ *	−	+ *

Sign of factor using test point

sign of function

* Intervals of > 0 solutions

$\boxed{\text{Ans: } (-\infty, 0) \cup (1, \infty)}$

$$\frac{4x+5}{x+2} \ge 3$$

① $$\frac{4x+5}{x+2} - 3 \ge 0$$

$$\frac{4x+5}{x+2} - \frac{3(x+2)}{x+2} \ge 0$$

$$\frac{4x+5-3(x+2}{x+2} \ge 0$$

$$\frac{4x+5-3x-6}{x+2} \ge 0$$

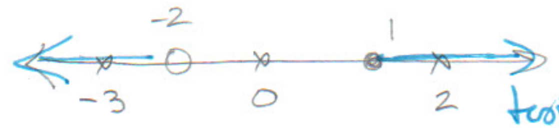
② $$\frac{x-1}{\underbrace{x+2}_{\neq 0}} \ge 0 \Rightarrow x = 1$$

$$x \neq -2$$

* solutions ≥ 0 intervals have dots

* denominator so pt. of discontinuity circle

intervals ≥ 0



	-3	0	2	test pts
$x-1$	$-$	$-$	$+$	sign of
$x+2$	$-$	$+$	$+$	factor
	$+$	$-$	$+$	sign of function

Ans $(-\infty, -2) \cup [1, \infty)$

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SUMMARY **Steps for Solving Polynomial and Rational Inequalities Algebraically**

Step 1: Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \qquad f(x) \ge 0 \qquad f(x) < 0 \qquad f(x) \le 0$$

For rational expressions, be sure that the left side is written as a single quotient and find the domain of f.

Step 2: Determine the real numbers at which the expression f equals zero and, if the expression is rational, the real numbers at which the expression f is undefined.

Step 3: Use the numbers found in Step 2 to separate the real number line into intervals.

Step 4: Select a number in each interval and evaluate f at the number.

(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.

(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.

If the inequality is not strict ($\ge$ or $\le$), include the solutions of $f(x) = 0$ that are in the domain of f in the solution set. Be careful to exclude values of x where f is undefined.