Precalculus

Lesson 000: Stuff You Should Already Know: Algebra II Review

Mrs. Snow, Instructor

Polynomials

A **polynomial** in one variable is an algebraic expression of the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants,* called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is a variable. If $a_n \neq 0$, it is the **leading coefficient**, and n is the **degree** of the polynomial.

$$-8x^3 + 4x^2 + 6x + 2$$

a) What is the degree of the polynomial?

b) What is the leading coefficient?

Special products

recognizing these products will speed up the math :)

Difference of Two Squares

$$(x-a)(x+a) = x^2 - a^2 \qquad \textbf{(2)}$$

Squares of Binomials, or Perfect Squares

$$(x+a)^2 = x^2 + 2ax + a^2 \qquad \textbf{(3a)}$$

$$(x-a)^2 = x^2 - 2ax + a^2 \qquad \textbf{(3b)}$$

Cubes of Binomials, or Perfect Cubes

$$(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3 \qquad \textbf{(4a)}$$

$$(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3 \qquad \textbf{(4b)}$$

Difference of Two Cubes

$$(x-a)(x^2 + ax + a^2) = x^3 - a^3 \qquad \textbf{(5)}$$

Sum of Two Cubes

$$(x+a)(x^2 - ax + a^2) = x^3 + a^3 \qquad \textbf{(6)}$$

Multiply the factors:

$(x-3)(x+3)$

$x^2 + \cancel{3x}$

$\quad - \cancel{3x} - 9$

$x^2 - 9$

*note difference of 2 squares

$(x+2)^2$

$(x+2)(x+2)$ (Binomial Square)

$x^2 + 4x + 4$

$(2x+1)(3x+4)$

$6x^2 + 8x$

$\quad + 3x + 4$

$6x^2 + 11x + 4$

$(x-2)(x^2+2x+4)$

$x^3 + \cancel{2x^2} + \cancel{4x}$

$\quad - \cancel{2x^2} - \cancel{4x} - 8$

$x^3 - 8$

note format:

$(a-b)(a^2+ab+b^2)$

Factor:

$x^4 - 16$

$x^2 - 4^2$

$(x+2)(x-2)$

$x^3 - 1$

$x^3 - 1^3$

$(x-1)(x^2 + x + 1)$

$9x^2 - 6x + 1$

$(9)(1)$
9
$1 \cdot 9$
$3 \cdot 3$
$-3 \cdot -3 = -6$

$9x^2 - 3x - 3x + 1 =$

$3x(x-1) - 3(x-1) =$

$(x-1)(3x-3)$

always factor out negative

$x^2 + 4x - 12$

$(x+6)(x-2)$

$3x^2 + 10x - 8$

$3(-8)$
-24
$1 \cdot 24$
$2 \cdot 12$
$3 \cdot 8$
$4 \cdot 6$
$-2 + 10$

$= 3x^2 + 12x - 2x - 8$

$3x(x+4) - 2(x+4) =$

$(x+4)(3x-2)$

$x^3 - 4x^2 + 2x - 8$

Group

$x^2(x-4) + 2(x-4) =$

$(x-4)(x^2+2)$

Simplifying Rational Expressions:

$$\frac{x^2 + 4x + 4}{x^2 + 3x + 2}$$

$$= \frac{\cancel{(x+2)}(x+2)}{\cancel{(x+2)}(x+1)} =$$

$$= \frac{x+2}{x+1}$$

$$\frac{x^3 - 8}{x^3 - 2x^2}$$

$$\frac{\cancel{(x-2)}(x^2+2x+2)}{x^2\cancel{(x-2)}} =$$

$$\frac{x^2+2x+2}{x^2}$$

$\frac{8-2x}{x^2-x-12}$ $= \frac{2(4-x)}{(x-4)(x+3)}$ — close but not the same factor out a(-1) $\frac{(2)(-1)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)} = \frac{-2}{x+3}$	

Multiplying and Dividing Rational Expressions

$\frac{x^2-2x+1}{x^3+x} \cdot \frac{4x^2+4}{x^2+x-2}$ $\frac{(x-1)\cancel{(x-1)}}{x\cancel{(x^2+1)}} \cdot \frac{4\cancel{(x^2+1)}}{(x+2)\cancel{(x-1)}}$ $= \frac{4(x-1)}{x(x+2)}$	$\dfrac{\frac{x+3}{x^2-4}}{\frac{x^2-x-12}{x^3-8}}$ $= \dfrac{\frac{x+3}{(x+2)(x-2)}}{\frac{(x+4)(x+3)}{(x-2)(x^2+2x+2)}} \div$ $\frac{\cancel{(x+3)}}{(x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x^2+2x+2)}{(x-4)\cancel{(x+3)}} =$ $= \frac{x^2+2x+2}{(x+2)(x-4)}$

Adding and Subtracting Rational Expressions

$\frac{x^2}{x^2-4} - \frac{1}{x}$ $(x+2)(x-2)$ $\frac{(x)}{(x)} \frac{x^2}{(x+2)(x-2)} - \frac{1}{x} \frac{(x+2)(x-2)}{(x+2)(x-2)}$ $= \frac{x^3-(x+2)(x-2)}{(x+2)(x-2)(x)}$	$\frac{x}{x^2+3x+2} + \frac{2x-3}{x^2-1}$ $\frac{x}{(x+2)(x+1)} \frac{(x-1)}{(x-1)} + \frac{(2x-3)}{(x+1)(x-1)} \frac{(x+2)}{(x+2)}$ $\frac{x(x-1)+(2x-3)(x+2)}{(x+2)(x+1)(x-1)} =$ $\frac{x^2-\cancel{x}+2x^2+\cancel{x}-6}{(x+2)(x+1)(x-1)} =$ $\frac{3x^2-6}{(x+2)(x+1)(x-1)}$

Simplify:

$$\frac{\frac{1}{2}+\frac{3}{x}}{\frac{x+3}{4}} \cdot \frac{4x}{4x} =$$

$$\frac{{}^{2}\cancel{4}x\left(\frac{1}{\cancel{2}}\right)+\frac{3}{\cancel{x}}(4x)}{\frac{\cancel{4}x(x+3)}{\cancel{4}}}$$

$$= \frac{2x+12}{x(x+3)}$$

Quadratic Formula

Quadratic Formula

Consider the quadratic equation

$$ax^2 + bx + c = 0 \quad a \neq 0$$

If $b^2 - 4ac < 0$, this equation has no real solution.
If $b^2 - 4ac \geq 0$, the real solution(s) of this equation is (are) given by the **quadratic formula.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the solutions, if any, of the equation:

$$3x^2 - 5x + 1 = 0$$

$a=3 \quad b=-5 \quad c=1$

$4ac =$
$4(3)(1)$

$$x = \frac{5 \pm \sqrt{25-12}}{6}$$

$$x = \frac{5 \pm \sqrt{13}}{6}$$

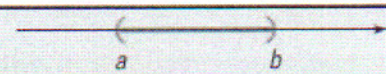
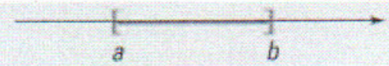
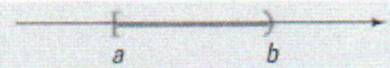
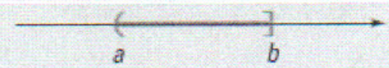
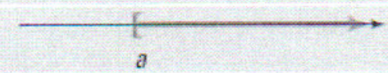
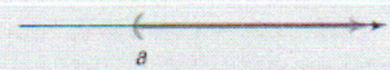
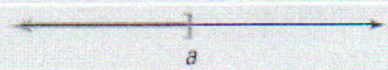
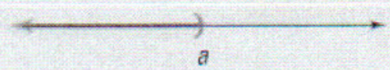
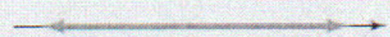
Interval Notation

Let a and b represent two real numbers with $a < b$.

A **closed interval,** denoted by $[a, b]$, consists of all real numbers x for which $a \le x \le b$.

An **open interval,** denoted by (a, b), consists of all real numbers x for which $a < x < b$.

The **half-open,** or **half-closed, intervals** are $(a, b]$, consisting of all real numbers x for which $a < x \le b$, and $[a, b)$, consisting of all real numbers x for which $a \le x < b$.

Interval	Inequality	Graph
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \le x \le b$	
The half-open interval $[a, b)$	$a \le x < b$	
The half-open interval $(a, b]$	$a < x \le b$	
The interval $[a, \infty)$	$x \ge a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \le a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

Write each inequality using interval notation

$1 \le x \le 3$

$[1, 3]$

$-4 < x < 0$

$(-4, 0)$

$x > 5$

$(5, \infty)$

$x \le 1$

$(-\infty, 1]$