Precalculus
Lesson 9.2 Graphs of Polar Equations
Mrs. Snow, Instructor

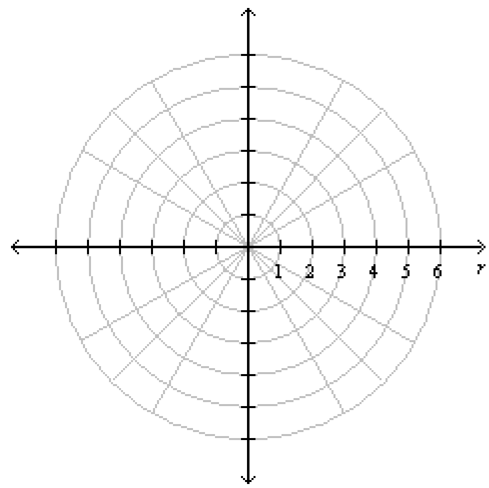
To plot points with polar coordinates, it is convenient to use a polar grid. It is sort of like the unit circle superimposed with graph paper, like below:

Special graphs:
θ = constant – graphs a line at angle θ
r=constant – graphs a circle of radius r

Sketch the graph of the equation and express the equation in rectangular coordinates:

$\theta = \frac{\pi}{3}$

$r = 3$

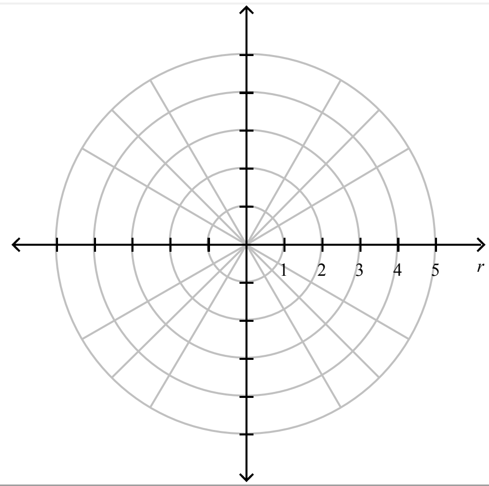


Graphing a Polar Equation of a Line:

Some equations can easily be expressed in rectangular coordinates. If this is the case then convert to rectangular coordinates.

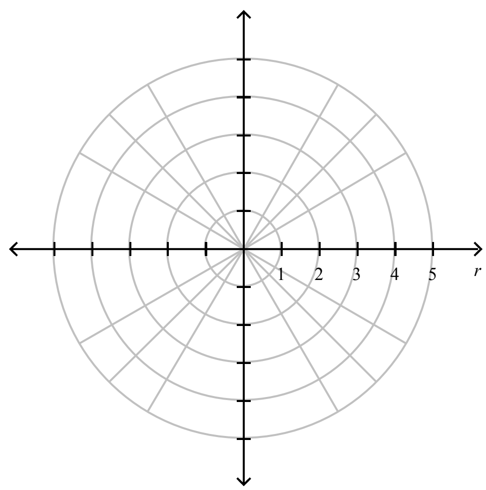
Identify and graph the equation

$$\theta = \frac{\pi}{4}$$



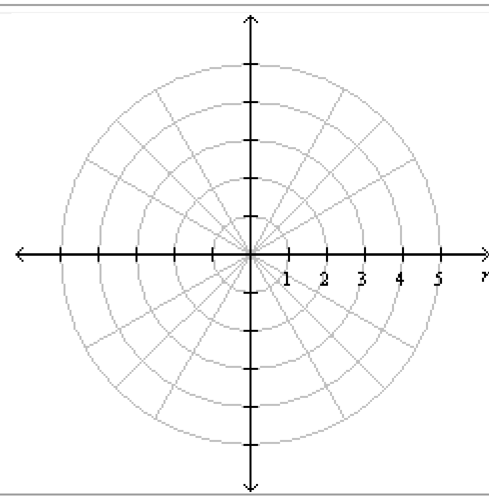
Identify and graph the equation

$$r \sin\theta = 2$$

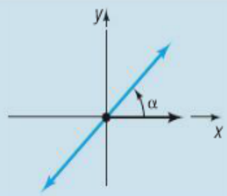
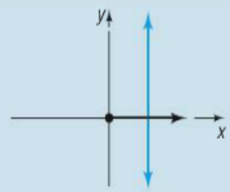
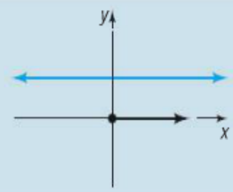


Identify and graph the equation

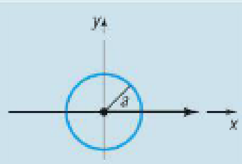
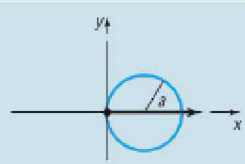
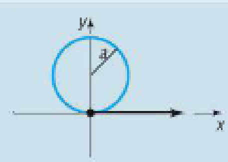
$$r \cos\theta = -3$$



In summary the equations in the forms below will graph as lines, note the forms for horizontal and vertical lines. Textbook (pg. 580:

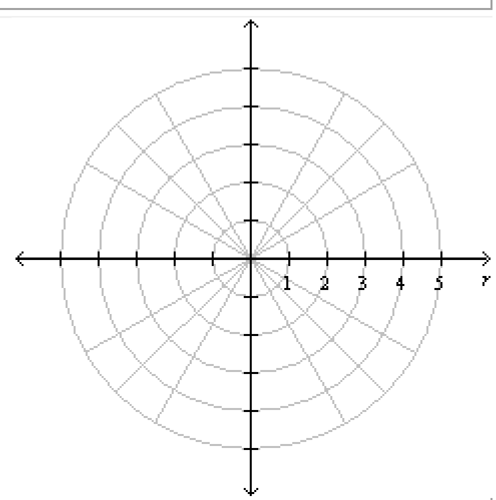
Lines			
Description	Line passing through the pole making an angle α with the polar axis	Vertical line	Horizontal line
Rectangular equation	$y = (\tan \alpha)x$	$x = a$	$y = b$
Polar equation	$\theta = \alpha$	$r \cos \theta = a$	$r \sin \theta = b$
Typical graph			

Identifying and Graphing a Polar Equation of a Circle (pg. 581):

Circles			
Description	Center at the pole, radius a	Passing through the pole, tangent to the line $\theta = \frac{\pi}{2}$, center on the polar axis, radius a	Passing through the pole, tangent to the polar axis, center on the line $\theta = \frac{\pi}{2}$, radius a
Rectangular equation	$x^2 + y^2 = a^2, \quad a > 0$	$x^2 + y^2 = \pm 2ax, \quad a > 0$	$x^2 + y^2 = \pm 2ay, \quad a > 0$
Polar equation	$r = a, \quad a > 0$	$r = \pm 2a \cos \theta, \quad a > 0$	$r = \pm 2a \sin \theta, \quad a > 0$
Typical graph			

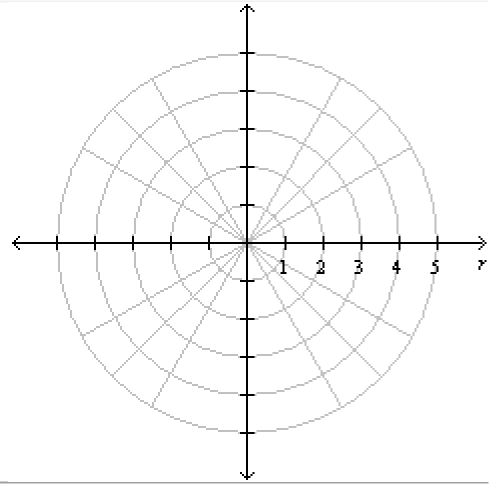
Sketch the polar equation (transform the equation into its rectangular form)

$$r = 4 \sin \theta$$

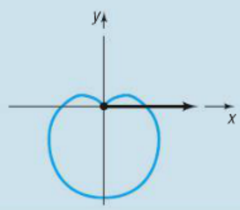
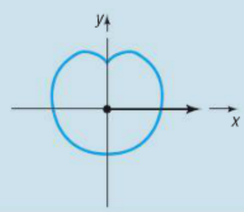
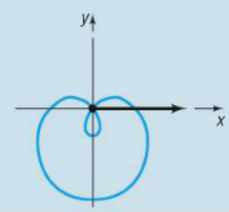


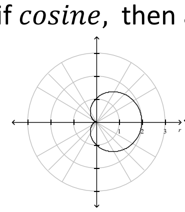
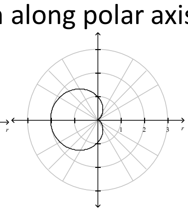
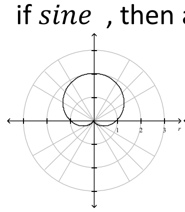
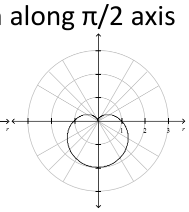
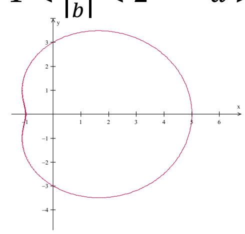
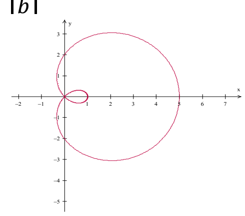
Sketch the polar equation

$$r = -2\cos\theta$$



Other Equations (pg. 581)

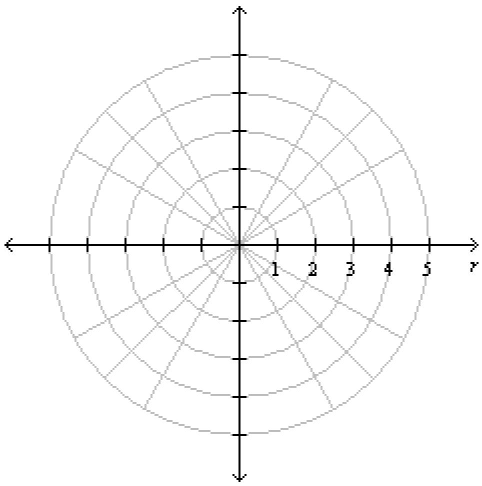
Other Equations			
Name	Cardioid	Limaçon without inner loop	Limaçon with inner loop
Polar equations	$r = a \pm a\cos\theta,\quad a > 0$	$r = a \pm b\cos\theta,\quad 0 < b < a$	$r = a \pm b\cos\theta,\quad 0 < a < b$
	$r = a \pm a\sin\theta,\quad a > 0$	$r = a \pm b\sin\theta,\quad 0 < b < a$	$r = a \pm b\sin\theta,\quad 0 < a < b$
Typical graph			

$\underline{a > 0,}$ $\underline{distance\ on\ axis\ is\ 2a}$ if $cosine$, then along polar axis   $r = 1 + \cos\theta$ $\quad r = 1 - \cos\theta$ if $sine$, then along π/2 axis   $r = 1 + \sin\theta$ $\quad r = 1 - \sin\theta$	if $cosine$: along polar axis if $sine$: along $\frac{\pi}{2}$ axis **a.** Limacon without inner loop if : $1 < \left\|\frac{a}{b}\right\| < 2 \quad \rightarrow a > b$  **b.** Limacon has an inner loop if: $\left\|\frac{a}{b}\right\| < 1 \quad \rightarrow a < b$ 

Cardioid – heart shaped (pg. 581)

graph $\boldsymbol{r = 1 - \sin\theta}$

$a = 1$; length of cardioid=__________ *equation is sine so along the* $\theta = \frac{\pi}{2}$ *axis and* $-\sin\theta$ *means … …*

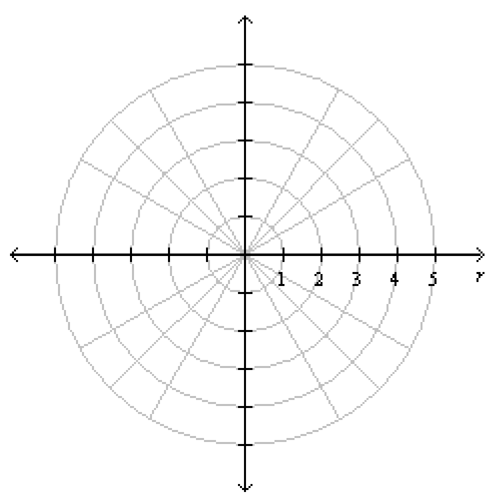


Whenever you cannot remember how to graph the polar equation, you can always graph a period of the trig function from $0 \leq \theta < 2\pi$ *and transfer the data over to a polar graph. Don't rely on memorizing an equation and associated graph shape, you will want a backup method!!*
Table of values *(use values for theta that yield friendly values for r):*

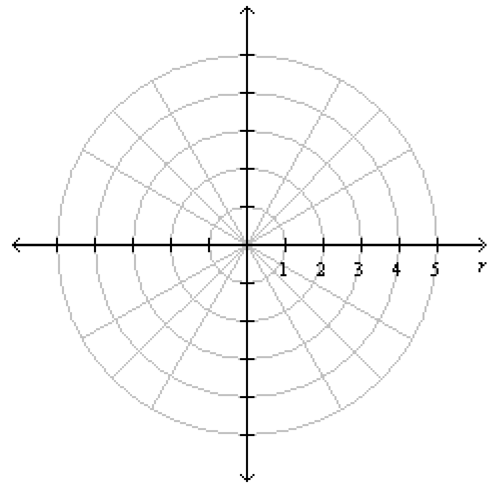
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$							
$r = 1 - \sin\theta$							

Graphing a limaçon without an inner loop

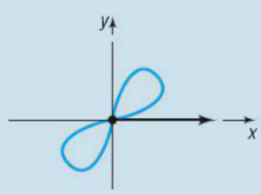
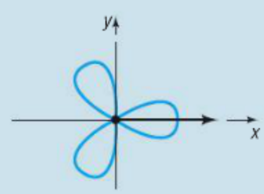
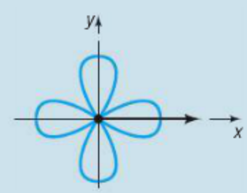
Sketch the graph of the equation
$r = 3 + 2\cos\theta$



Graphing a limaçon with an inner loop

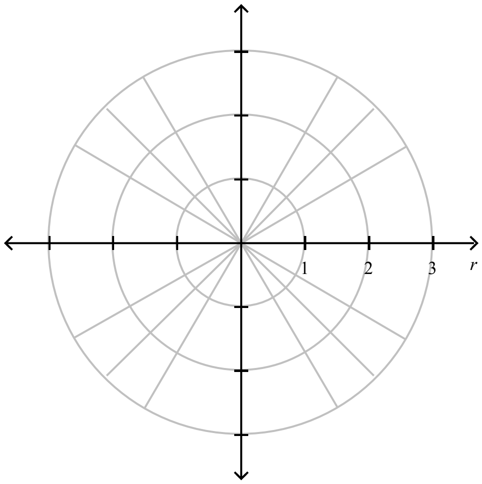
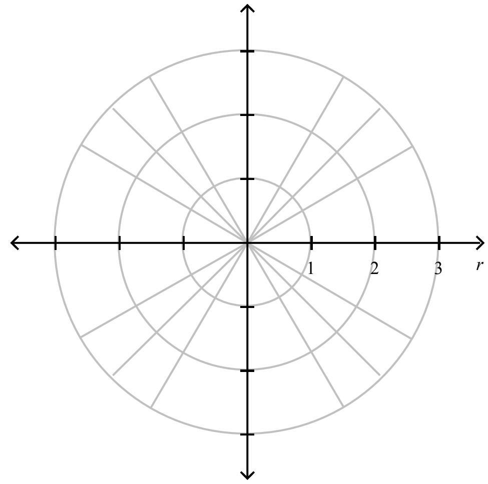
$r = 1 + 2\cos\theta$	

More Equations

Name	Lemniscate	Rose with three petals	Rose with four petals
Polar equations	$r^2 = a^2\cos(2\theta), \quad a > 0$	$r = a\sin(3\theta), \quad a > 0$	$r = a\sin(2\theta), \quad a > 0$
	$r^2 = a^2\sin(2\theta), \quad a > 0$	$r = a\cos(3\theta), \quad a > 0$	$r = a\cos(2\theta), \quad a > 0$
Typical graph			

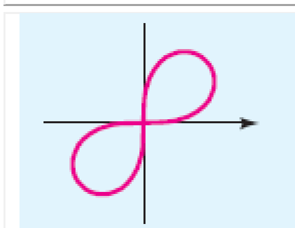
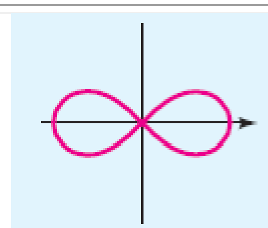
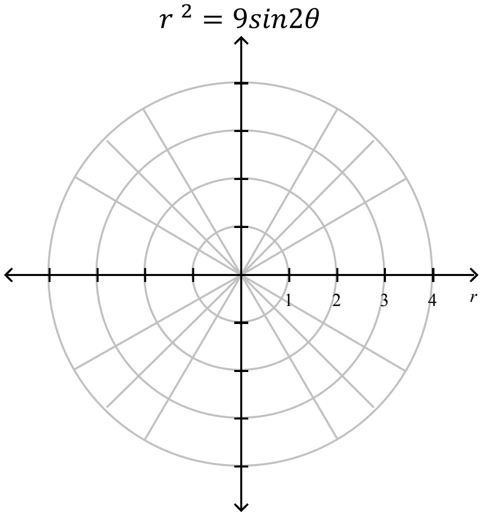
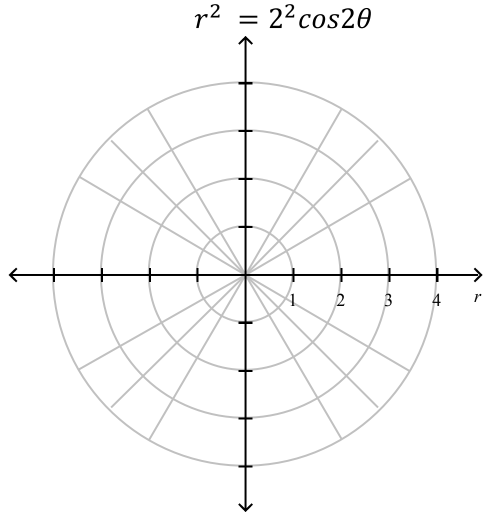
$a = petal\ length$	number of petals look at coefficient of θ: $\begin{cases} odd = n\ petals \\ even = 2n\ petals \end{cases}$ $a = length\ of\ petal$

Graphing a Polar Equation: n-leaved rose (petals)

$r = 2\sin 3\theta$	$r = 2\cos 2\theta$
	

Lemniscates – Figure 8 shaped curves

$\boldsymbol{r^{\,2} = a^2 sin2\theta}$ $\boldsymbol{r^2 = a^2 cos2}$ $\boldsymbol{a = petal\ length}$

 $r^2 = a^2 \sin 2\theta$	 $r^2 = a^2 \cos 2\theta$
$r^{\,2} = 9sin2\theta$ 	$r^2 = 2^2cos2\theta$ 

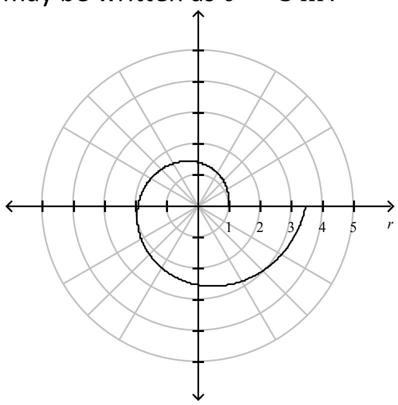
Graphing a Polar Equation (spiral)

It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

There are several equations that will produce a spiral. The **logarithmic spiral**

$$r = e^{\theta/5}$$

may be written as $\theta = 5\ln r$



Archimedes Spiral is in the form of

$$r = a\theta$$