Precalculus
Lesson 8.5: Simple Harmonic Motion and combining Waves
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Repetitive motion as seen in the rising and falling of the tide, the changing of the seasons year after year, or sunrise and sunset are all examples of periodic behavior. Trigonometric functions are ideally suited for modeling periodic behavior.

cycle: one complete vibration or motion of an object is a cycle

amplitude: $|a|$ the distance from the object's rest position to its point of greatest displacement

period: $\dfrac{2\pi}{\omega}$ time required to complete one complete cycle

frequency: $f = \dfrac{\omega}{2\pi}$ number of cycles per unit of time

reciprocals!!

simple harmonic motion: an object is considered to be in simple harmonic motion if the displacement **d** of an object at time **t** can be modeled by:

$$d = a\,\sin\omega t \quad or \quad d = a\cos\omega t$$

Thoughts: consider the equation that models harmonic motion at time 0, the starting point.

$$d = a\,\sin\omega t \quad at\ t = 0$$
$$d = a\sin 0 = 0 \quad or\ at\ time = 0, \quad displacement = 0$$

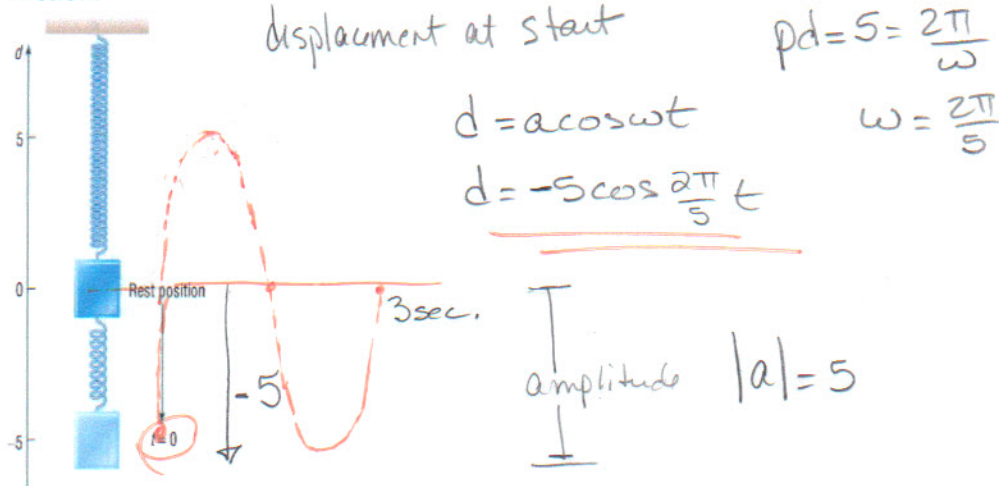$$d = a\cos\omega t \quad at\ time = 0$$
$$d = a\cos 0 \quad or\ at\ time = 0, \quad displacement = a$$

So, one equation works for situations where there is no displacement at initial conditions and the other equations is applicable for initial conditions that have a displacement.

Build a Model for an Object in Harmonic Motion: a vibrating spring

Suppose that an object attached to a coiled spring is pulled down a distance of 5 inches from its rest position and then released. If the time from one oscillation is 3 seconds, develop a model that relates the displacement d of the object from its rest position after time t (in seconds). Assume no friction.

displacement at start

$d = a\cos\omega t$

$d = -5\cos\dfrac{2\pi}{5}t$

3sec.

amplitude $|a| = 5$

$pd = 5 = \dfrac{2\pi}{\omega}$

$\omega = \dfrac{2\pi}{5}$

Suppose that the displacement d (in meters) of an object at time t (in seconds) satisfies the equation $d = 10\sin(5t)$

a) Describe the motion of the object
b) What is the maximum displacement from its resting position
c) What is the time required of an oscillation?
d) What is the frequency?

$$d = a \sin \omega t$$

$$(\text{time}) \quad P = \frac{2\pi}{\omega} = \frac{2\pi}{5} \text{ sec}$$

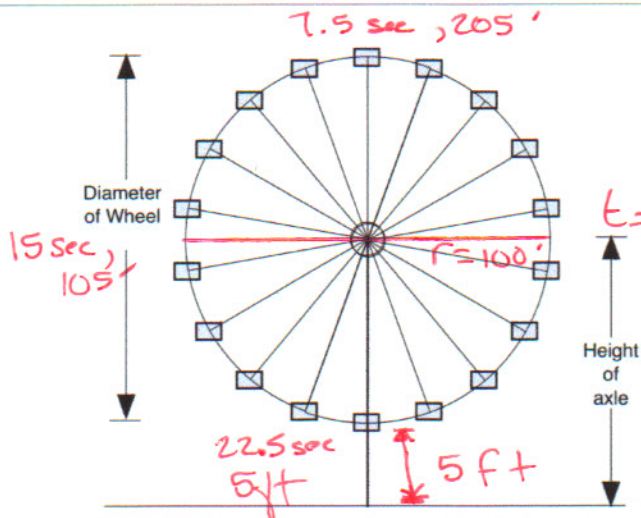$$f = \frac{1}{P} = \frac{5}{2\pi} \frac{osc.}{sec}$$

(a) simple harmonic motion
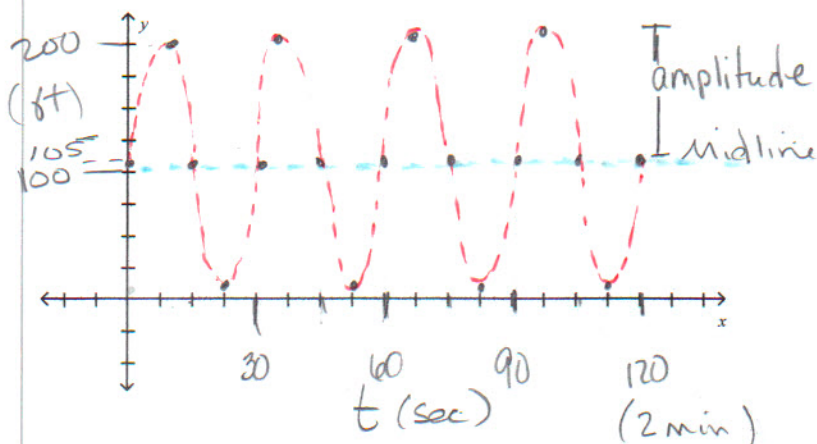
(b) 10 meters (amplitude)

(c) $\frac{2\pi}{5}$ sec.

(d) $\frac{5}{2\pi}$ osc/sec.

← Period

A person is seated on a Ferris wheel of radius 100 ft that makes one rotation every 30 s. The center of the wheel is 105 ft above the ground. Find and graph a function to represent the person's height above the ground at any time t of a 2-min ride. Assume uniform speed from the beginning to the end of the ride and that the person is at the level of the center of the wheel and headed up when the ride begins. (hint: Make a table of time vs. height, graph and build an equation based on the graph.)

7.5 sec, 205'

Diameter of Wheel

15 sec, 105'

r = 100'

Height of axle

t = 0, 105 ft
30 sec, 105 ft

22.5 sec
5 ft

5 ft

t (sec)	h (ft)
0	105
7.5	205
15	105
22.5	5
30	105

amplitude

midline

So amplitude = radius

$a = 100$

$$pd: 30 = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{30}$$

$$\omega = \pi/15$$

200 (ft)

105
100

30   60   90   120
t (sec)   (2 min)

looks like a sine graph, but shifted up 105 ft

$$d = a \sin \omega t + 105$$

$$d = 100 \sin \frac{\pi}{15} t + 105$$