

Precalculus
Lesson 8.3: Law of Cosines
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Last section we looked at the law of sines. There are two other situations where the law of sines will not work; here we will use the **Law of Cosines**:

Case 3 – Two sides and the angle included between the two sides are known (SAS).

Case 4 – Three sides are known (SSS).

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

EXAMPLE: SSS

The sides of a triangle are: $a = 3, b = 4,$ and $c = 6$. Find the angles of the triangle

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$3^2 = 4^2 + 6^2 - 2(4)(6) \cos A$$

$$9 = 16 + 36 - 2(4)(6) \cos A$$

$$\cos A \approx .8958$$

$$A \approx \underline{26.4^\circ}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$16 = 9 + 36 - 2(3)(6) \cos B$$

$$\cos B \approx .8055$$

$$B \approx \underline{36.3^\circ}$$

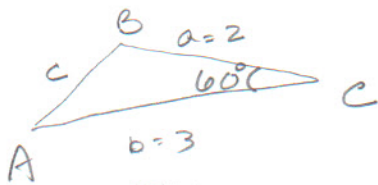
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$36 = 9 + 16 - 2(3)(4) \cos C$$

$$\cos C \approx -.4583$$

$$C \approx \underline{117.3^\circ}$$

SAS: Solve the triangle ABC, where $\angle C = 60^\circ$, $a = 2$, and $b = 3$



$$\begin{aligned} A &= 40.9^\circ \\ B &= 79.1^\circ \\ C &= \sqrt{7} \end{aligned}$$

Find c :

$$c^2 = 4 + 9 - 2(2)(3) \cos 60^\circ$$

$$c^2 = 7 \quad c = \sqrt{7}$$

careful!

Find A

$$4 = 9 + 7 - 2(3)(\sqrt{7}) \cos A$$

$$\cos A \sim .755 \rightarrow$$

$$A \sim 40.9^\circ$$

Find B :

$$9 = 4 + 7 - 2(2)(\sqrt{7}) \cos B$$

$$\cos B \sim .188 \rightarrow$$

$$B \sim 79.1^\circ$$

You may find B : $B = 180 - A - C$

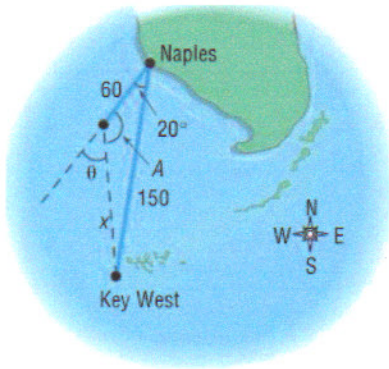
But what if A is wrong??

(Good way to check)

Navigation

A motorized sail boat leaves Naples, Florida bound for Key West, 150 miles away. Maintaining a constant speed of 15 mph, but encountering heavy crosswinds and strong currents, the crew finds after 4 hours that the sailboat is off course by 20° .

- How far is the sailboat from Key West at this time? b
- Through what angle should the sailboat turn to correct its course θ
- How much time has been added to the trip because of this? Assume a constant speed of 15 mph.



$$b^2 = 150^2 + 60^2 - 2(150)(60) \cos 20^\circ$$

$$b^2 = 9185.0 \dots$$

$$b \sim 95.8 \text{ mi from Key West}$$

$$\theta = 180 - A \quad \text{96 mi}$$

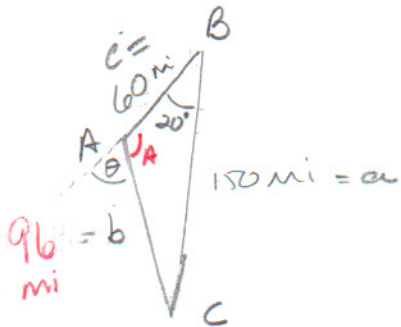
know 3-sides

$$150^2 = 60^2 + 96^2 - 2(60)(96) \cos A$$

$$A \sim 147.2^\circ$$

$$\theta = 180 - 147.2$$

$$\theta = \underline{\underline{32.8^\circ}}$$



$$15 \frac{\text{mi}}{\text{hr}} (4 \text{ hr}) = 60 \text{ mi}$$