Precalculus
Lesson 7.6: Double-angle and Half-angle Formulas
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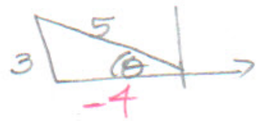
And more identities……

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\cos(2\theta) = 1 - 2\sin^2\theta$$
$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Find the exact value using the double-angle formulas:

Given $\sin\theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$ Q II



3-4-5 Δ

$\cos\theta = \frac{A}{H} = \frac{-4}{5}$

$\tan\theta = \frac{O}{A} = \frac{3}{-4}$

$\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right)$

$= \frac{-24}{25}$

$\cos 2\theta$

#1 $= \cos^2\theta - \sin^2\theta$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25}$

$= \boxed{\frac{7}{25}}$

#2 $1 - 2\sin^2\theta = 1 - 2\left(\frac{3}{5}\right)^2$

$= \frac{25}{25} - \frac{18}{25} = \boxed{\frac{7}{25}}$

#3 $2\cos^2\theta - 1 = 2\left(\frac{16}{25}\right) - 1$

$= \frac{32}{25} - \frac{25}{25} = \boxed{\frac{7}{25}}$

$\tan 2\theta$

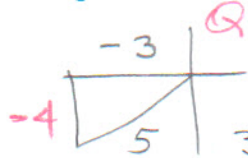
$= \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{3}{\cancel{4}2}\right)}{\frac{16}{16} - \frac{9}{16}}$

$= \frac{-\frac{3}{2}}{\frac{7}{16}}$

$= -\frac{3}{2} \cdot \frac{\cancel{16}8}{7} = \boxed{\frac{-24}{7}}$

Given: $\cos\theta = -\frac{3}{5}$, $\pi < \theta < \frac{3\pi}{2}$

Q III



3-4-5 Δ

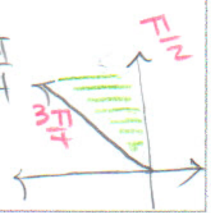
$\sin\theta = \frac{O}{H} = \frac{-4}{5}$

$\pi < \theta < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{2 \cdot 2}$

$\therefore$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$



$\frac{\theta}{2}$ in Quad II

$\sin\frac{\theta}{2}$

$= \pm\sqrt{\frac{1-\left(\frac{-3}{5}\right)}{2}}$

$= \sqrt{\frac{\frac{5}{5}+\frac{3}{5}}{2}}$

$= \sqrt{\frac{\frac{\cancel{8}^{4}}{5}}{\cancel{2}_{1}}}$

$= \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$= \boxed{\frac{2\sqrt{5}}{5}}$

$\cos\frac{\theta}{2}$

$= -\sqrt{\frac{1+\left(\frac{-3}{5}\right)}{2}} =$

$= -\sqrt{\frac{\frac{5}{5}-\frac{3}{5}}{2}} =$

$= -\sqrt{\frac{\frac{\cancel{2}}{5}}{\cancel{2}}} = -\sqrt{\frac{1}{5}}$

$= -\frac{1}{\sqrt{5}}$

$= \boxed{\frac{-\sqrt{5}}{5}}$

$\tan\frac{\theta}{2}$

$\frac{1-\cos\theta}{\sin\theta} = \frac{1-\left(\frac{-3}{5}\right)}{\frac{-4}{5}}$

$= \frac{\frac{5}{5}+\frac{3}{5}}{\frac{-4}{5}} = \frac{\frac{\cancel{8}^{2}}{\cancel{5}_{1}}}{\frac{-\cancel{4}_{1}}{\cancel{5}_{1}}} = \boxed{-2}$

For a good value lesson, extra problems !!!...

#1

$\sin\left(\underbrace{2\sin^{-1}\tfrac{1}{2}}\right) \Rightarrow \sin\left(\cancel{2}\left(\frac{\pi}{\cancel{6}\,3}\right)\right)$

$=$ $\quad \sin^{-1}\frac{1}{2} = \theta \qquad = \sin\frac{\pi}{3}$

$\sin\theta = \frac{1}{2} \qquad = \boxed{\frac{\sqrt{3}}{2}}$

$\theta = \frac{\pi}{6}$

#2

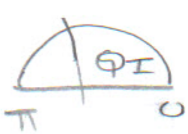
Substitute θ as $\cos^{-1}\frac{4}{5}$

$\sin\left(\underbrace{2\cos^{-1}\tfrac{4}{5}}\right) = \sin(2\theta) = 2\sin\theta\cos\theta$

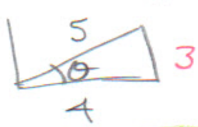
$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\cos^{-1}\frac{4}{5} = \theta$

$\cos\theta = \frac{4}{5}$



$= \boxed{\frac{24}{25}}$



$\sin\theta = \frac{O}{H} = \frac{3}{5}$