Precalculus
Lesson 7.5: Sum and Difference Formulas
Mrs. Snow, Instructor

We continue with our study of more trigonometric identities:

Sum and Difference Formlas

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

The sum and difference identities may be used to find the exact value of angles not found on the unit circle.

$\cos 75°$

$30°$
$45°$
$75°$

$= \cos(30+45)$

$= \cos 30 \cos 45 - \sin 30 \sin 45$

$= \frac{\sqrt3}{2}\ \frac{\sqrt2}{2} - \frac12\ \frac{\sqrt2}{2}$

$= \frac{\sqrt6}{4} - \frac{\sqrt2}{4}$

$= \frac{\sqrt6-\sqrt2}{4} = \frac14\left(\sqrt6-\sqrt2\right)$

for Online HW

$\cos\frac{\pi}{12}$ $\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ $\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)$

$= \cos\frac{4\pi}{12}\cos\frac{3\pi}{12} + \sin\frac{4\pi}{12}\sin\frac{3\pi}{12}$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac12\cdot\frac{\sqrt2}{2} + \frac{\sqrt3}{2}\cdot\frac{\sqrt2}{2}$

$= \frac{\sqrt2}{4} + \frac{\sqrt6}{4} = \frac14\left(\sqrt2+\sqrt6\right)$

$\sin\frac{7\pi}{12}$ $\frac{3\pi}{12}+\frac{4\pi}{12}$

$= \sin\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right) =$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt2}{2}\cdot\frac12 + \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2}$

$= \frac{\sqrt2}{4} + \frac{\sqrt6}{4} = \frac14\left(\sqrt2+\sqrt6\right)$

$\sin 80° \cos 20° - \cos 80° \sin 20°$

$\sin(\alpha-\beta)$

$= \sin(80-20)$

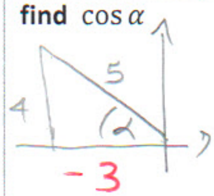
$= \sin 60$

$= \frac{\sqrt3}{2}$

Given: $\sin\alpha = \frac{4}{5}$, $\pi < \alpha < \frac{\pi}{2}$ Quad II and

$\sin\beta = -\frac{2\sqrt{5}}{5}$, $\pi < \beta < \frac{3\pi}{2}$ Quad III

find $\cos\alpha$



$\sin\alpha = \frac{4}{5} = \frac{O}{H}$

3-4-5 △

$x^2 + 16 = 25$

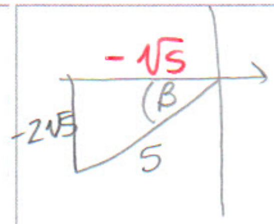
$x^2 = 9$

$x = 3$

Q II

$\cos\alpha = \frac{A}{H} = \boxed{\frac{-3}{5}}$

$\cos\beta$



$\sin\beta = \frac{-2\sqrt{5}}{5}$

$20 + x^2 = 25$

$x^2 = 5$

$x = \sqrt{5}$

Q III

$\cos\beta = \frac{A}{H}$

$= \boxed{\frac{-\sqrt{5}}{5}}$

$\cos(\alpha+\beta)$ $= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-3}{5}\right)\left(-\frac{\sqrt{5}}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{-2\sqrt{5}}{5}\right)$

$= \frac{3\sqrt{5}}{25} + \frac{8\sqrt{5}}{25}$

$= \boxed{\frac{11\sqrt{5}}{25}}$

$\sin(\alpha+\beta)$ $= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(\frac{-\sqrt{5}}{5}\right) + \left(\frac{-3}{5}\right)\left(\frac{-2\sqrt{5}}{5}\right)$

$= \frac{-4\sqrt{5}}{25} + \frac{6\sqrt{5}}{25}$

$= \boxed{\frac{2\sqrt{5}}{25}}$

Establish the identities (prove):

$$\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta} = \cot\alpha\cot\beta + 1$$

LHS

$$= \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$

make into 2 fractions

$$= \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$

$$= \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + 1$$

$= \text{RHS}$ Q.E.D.

$$\tan(\theta+\pi) = \tan\theta$$

LHS

$$= \frac{\sin(\theta+\pi)}{\cos(\theta+\pi)}$$

$$= \frac{\sin\theta\,\overset{1}{\cancel{\cos\pi}} + \cos\theta\,\overset{0}{\cancel{\sin\pi}}}{\cos\theta\,\underset{1}{\cancel{\cos\pi}} - \sin\theta\,\underset{0}{\cancel{\sin\pi}}}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$= \tan\theta$

$= \text{RHS}$

Q.E.D.

$$\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$$

LHS

$$= \frac{\sin\left(\theta + \frac{\pi}{2}\right)}{\cos\left(\theta + \frac{\pi}{2}\right)}$$

$$= \frac{\sin\theta \cancel{\cos\frac{\pi}{2}}^{\,0} + \cos\theta \cancel{\sin\frac{\pi}{2}}^{\,1}}{\cos\theta \cancel{\cos\frac{\pi}{2}} \to 0 - \sin\theta \cancel{\sin\frac{\pi}{2}} \to 1}$$

$$= \frac{\cos\theta}{-\sin\theta}$$

$$= -\cot\theta$$

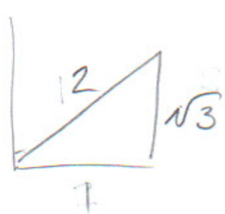
$= \text{RHS}$ QED :)

Find the exact value of:

$$\sin\left(\underbrace{\boxed{\cos^{-1}\frac{1}{2}}}_{\alpha} + \underbrace{\sin^{-1}\frac{3}{5}}_{\beta}\right)$$

$\cos\alpha = \frac{1}{2}$ $\quad 0 \le \alpha \le \pi$

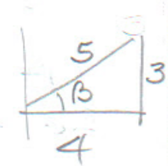
$= \frac{A}{H}$



$\sin\alpha = \frac{\sqrt{3}}{2}$

$\sin\beta = \frac{3}{5}$ $\quad -\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$

$= \frac{O}{H}$



$\cos\beta = \frac{4}{5}$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{\sqrt{3}}{2}\left(\frac{4}{5}\right) + \frac{1}{2}\left(\frac{3}{5}\right)$$

$$= \frac{4\sqrt{3} + 3}{10}$$