Precalculus
Lesson 7.3: Trigonometric Equations
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We have studied trigonometric graphs and expressions. The next skill we need to learn is how to solve trigonometric equations.

Determine whether or not $\theta = \frac{\pi}{4}$ is a solution for the equation below.

$\theta = \frac{\pi}{4}$

$2\sin\theta - 1 = 0$

$2\sin\frac{\pi}{4} - 1 \stackrel{?}{=} 0$

$2\left(\frac{\sqrt{2}}{2}\right) - 1 \stackrel{?}{=} 0$

$\sqrt{2} - 1 \neq 0$

We can solve linear trigonometric equations:

$$2\sin\theta + \sqrt{3} = 0, \quad \underbrace{0 \le \theta \le 2\pi}_{\text{over interval}}$$

$2\sin\theta = -\sqrt{3}$

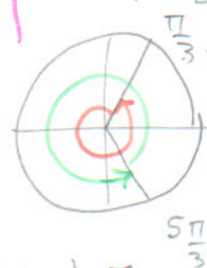
$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = \left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

Solve for θ.
Give a general formula for all the solutions.
List 8 of the solutions.

$$\cos\theta = \frac{1}{2}$$

$\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$ over pd $[0, 2\pi]$



for every 2π rotation we get another solution so:

General solution:

$$\boxed{\begin{aligned}\theta &= \frac{\pi}{3} + 2k\pi \\ \theta &= \frac{5\pi}{3} + 2k\pi\end{aligned}}$$

8 solutions:

$\theta = \frac{\pi}{3} + 2\pi$ $\qquad$ $\theta = \frac{5\pi}{3} + 2\pi$

$\theta = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$ $\qquad$ $\theta = \frac{5\pi}{3} + \frac{6\pi}{3} = \frac{11\pi}{3}$

$\theta = \frac{7\pi}{3} + \frac{6\pi}{3} = \frac{13\pi}{3}$ $\qquad$ $\theta = \frac{11\pi}{3} + \frac{6\pi}{3} = \frac{17\pi}{3}$

$\theta = \frac{13\pi}{3} + \frac{6\pi}{3} = \frac{19\pi}{3}$ $\qquad$ $\theta = \frac{17\pi}{3} + \frac{6\pi}{3} = \frac{23\pi}{3}$

Solve: horiz. compression

$$\sin(2\theta) = \frac{1}{2}, \quad 0 \le \theta \le 2\pi$$

where is sine $= \frac{1}{2}$?

So:

$2\theta = \frac{\pi}{6}$ & $\frac{5\pi}{6}$

Put into general form:

$\frac{1}{2}(2\theta) = \left(\frac{\pi}{6} + 2k\pi\right)\frac{1}{2}$

$\theta = \frac{\pi}{12} + k\pi$

And

$\frac{1}{2}(2\theta) = \left(\frac{5\pi}{6} + 2k\pi\right)\frac{1}{2}$

$\theta = \frac{5\pi}{12} + 2k\pi$

θ over interval $0 \le \theta \le 2\pi$

$\theta = \frac{\pi}{12}$ $(k=0)$ $\quad \theta = \frac{5\pi}{12}$

$\theta = \frac{\pi}{12} + 1\pi$
$= \frac{\pi}{12} + \frac{12\pi}{12}$
$= \frac{13\pi}{12}$

$\theta = \frac{5\pi}{12} + 1\pi$
$= \frac{5\pi}{12} + \frac{12\pi}{12}$
$= \frac{17\pi}{12}$

$\theta = \frac{\pi}{12} + 2\pi$
$= \frac{\pi}{12} + \frac{24\pi}{12}$
$= \cancel{\frac{25\pi}{12} > 2\pi}$

$\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

Solve:

$$\tan\left(\theta - \frac{\pi}{2}\right) = 1, \quad 0 \le \theta \le 2\pi$$

$\tan = 1$ at $\frac{\pi}{4}$ (pd of tan is π)

$\theta - \frac{\pi}{2} = \frac{\pi}{4} + k\pi$

$\theta = \frac{\pi}{4} + \frac{\pi}{2} + k\pi$

$\theta = \frac{3\pi}{4} + k\pi$

now over interval $0 \le \theta \le 2\pi$

$\theta = \frac{3\pi}{4}$ $(k=0)$

$\theta = \frac{3\pi}{4} + \pi$
$= \frac{7\pi}{4}$

$\cancel{\theta = \frac{3\pi}{4} + 2\pi}$

$\theta = \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

Solving a trigonometric quadratic equation:

$$2sin^2\theta - 3sin\theta + 1 = 0, \quad 0 \le \theta \le 2\pi$$

factor!

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0 \qquad \sin\theta - 1 = 0$

$2\sin\theta = 1 \qquad \sin\theta = 1$

$\sin\theta = \frac{1}{2} \qquad \theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$

Solving with trigonometric identities:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$

$$3\cos\theta + 3 = 2\sin^2\theta\ ,\ \ 0 \le \theta \le 2\pi$$

$$3\cos\theta + 3 - 2\sin^2\theta = 0$$
$$3\cos\theta + 3 - 2(1-\cos^2\theta) = 0$$
$$3\cos\theta + 3 - 2 + 2\cos^2\theta = 0$$
$$2\cos^2\theta + 3\cos\theta + 1 = 0$$
$$(2\cos\theta + 1)(\cos\theta + 1) = 0$$

$2\cos\theta + 1 = 0$ $\qquad$ $\cos\theta + 1 = 0$

$2\cos\theta = -1$ $\qquad$ $\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$ $\qquad$ $\theta = \pi$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$\theta = \left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta + \sin\theta = 2\ ,\ \ 0 \le \theta \le 2\pi$$

$$1 - \sin^2\theta + \sin\theta = 2$$
$$1 - \sin^2\theta + \sin\theta - 2 = 0$$
$$-1(-\sin^2\theta + \sin\theta - 1) = 0$$
$$\sin^2\theta - \sin\theta + 1 = 0$$

+ cannot factor

+ discriminant?

$b^2 - 4ac$

$1 - 4(1)(1) = $ negative

$\therefore$ no real solution

so: $\emptyset$ empty set

Graphing utilities are always nice.... solve, rounding to two decimal places.

$$5\sin x + x = 3$$

$$5\sin x + x - 3 = 0$$
$$y_1 = 5\sin x + x - 3 = 0$$

Zeros are solutions:

$$x = \{.52, 3.18, 5.71\}$$

$$\tan\theta = -2\ ,\ \ 0 \le \theta \le 2\pi$$

$$\tan\theta + 2 = 0$$

$$\theta = \{2.03, 5.18\}$$