Precalculus
Lesson 7.2: The Inverse Trigonometric Functions (continued)
Mrs. Snow, Instructor

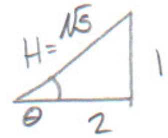
Composing a Trig Function
What???? Evaluate a trig function involving inverse functions.

Find the exact value of :

$$\sin\left(tan^{-1}\frac{1}{2}\right) = \sin\theta$$

$\tan^{-1}\frac{1}{2} = \theta$

$\tan\theta = \frac{1}{2} = \frac{O}{A}$



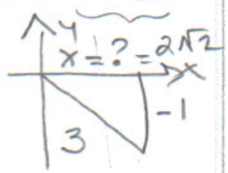
$\Rightarrow \sin\theta = \frac{O}{H} = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

1. Let θ equal the inverse function
2. By definition: $\theta = tan^{-1}\frac{1}{2}$ $\therefore \tan\theta = \frac{1}{2}$
3. Set up a triangle in which $\tan\theta = \frac{1}{2}$

$$\cos\left[sin^{-1}\left(-\frac{1}{3}\right)\right] = \cos\theta$$

$\sin^{-1} -\frac{1}{3} = \theta$ Interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\sin\theta = -\frac{1}{3} = \frac{O}{H}$



$\cos\theta = \frac{A}{H} = \boxed{\frac{2\sqrt{2}}{3}}$

$x^2 + 1 = 9$

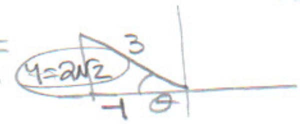
$x^2 = 8$

$x = 2\sqrt{2}$

$$\tan\left[cos^{-1}\left(-\frac{1}{3}\right)\right] = \tan\theta$$

$\cos^{-1} -\frac{1}{3} = \theta$ Int $[0, \pi]$

$\cos\theta = -\frac{1}{3} = \frac{A}{H}$



$\tan\theta = \frac{O}{A} = \frac{2\sqrt{2}}{-1}$

$= \boxed{-2\sqrt{2}}$

$y^2 + 1 = 9$

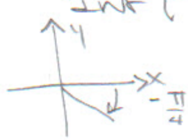
$y^2 = 8$

$y = 2\sqrt{2}$

$$cos^{-1}\left[\underbrace{\tan\left(-\frac{\pi}{4}\right)}\right] =$$

Int $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan -\frac{\pi}{4} = -1$



$-\frac{\pi}{4} = \theta$

$\cos^{-1} -1 = ?$ $0 \le x \le \pi$

$\cos\theta = -1$ $\boxed{\theta = \pi}$

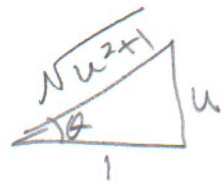
Write a Trigonometric expression as an Algebraic Expression:

Look back to the first example. What is the first step?

$$\sin(tan^{-1} u) = \sin\theta$$

$\tan^{-1} u = \theta$

$\tan\theta = \frac{u}{1} = \frac{o}{a}$



So :

$1^2+u^2 = h^2$

$\sqrt{1+u^2} = hyp$

$\sin\theta = \frac{o}{H} = \frac{u}{\sqrt{u^2+1}} \cdot \frac{\sqrt{u^2+1}}{\sqrt{u^2+1}}$

Rationalize denominator

$$\boxed{= \frac{u\sqrt{u^2+1}}{u^2+1}}$$