Precalculus

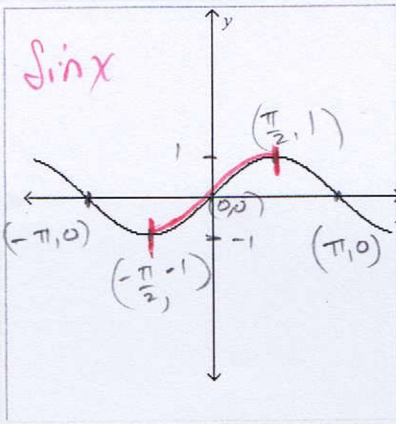
Lesson 7.1: The Inverse Sine, Cosine and Tangent Functions

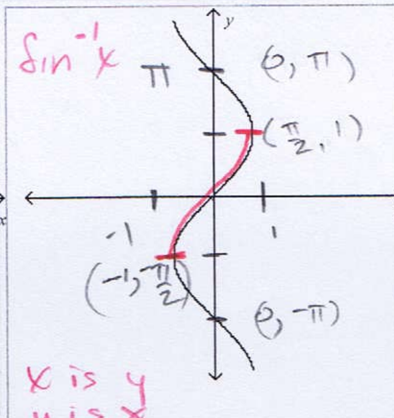
Mrs. Snow, Instructor

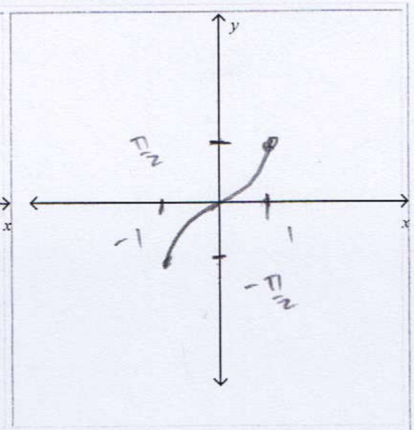
in Section 5.2 we discussed inverse functions. If a function is one-to-one if has an inverse. We are able to restrict the domain of a function to make it one-to-one. While trig functions are of course functions, they are not 1 – 1, so they do not have inverses. We can, however, force our trig functions into being 1 – 1 by limiting their domain.

Inverse: x is y & y is x

Inverse Sine Function: sin^{-1} *is also known as* arcsine *and written as* arcsin







$\sin x = y$	$\sin^{-1} x = y$	$\sin^{-1} x = y$
Domain: $(-\infty, \infty)$	D $[-1, 1]$	Domain: $[-1, 1]$
Range: $[-1, 1]$	R $(-\infty, \infty)$	Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

By definition: the inverse sine function is the function sin^{-1}, with a domain of $[-1, 1]$ and range of $[-\frac{\pi}{2}, \frac{\pi}{2}]$ defined by:

$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$

$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

sin x = y
switch for f^{-1}
→ sin y = x
&
$y = \sin^{-1} x$

Finding the exact value of an inverse sine function:

$sin^{-1}\, 1$	$sin^{-1} -\frac{1}{2}$	$sin^{-1} \frac{3}{2}$
Sine of what θ is 1: $\sin \theta = 1$ $\theta = \frac{\pi}{2}$ look at graph w/ restricted the domain	$\sin \theta = -\frac{1}{2}$ $\theta = -\frac{\pi}{6}$	$\sin \theta = \frac{3}{2}$?? undefined $\sin \theta =$ max 1 min -1

Approximate values of inverse sine functions may be found using a calculator. Express the answer in radians rounded to 2 decimal places.

$sin^{-1}\frac{1}{3}$ $\approx .34$ rad	$sin^{-1}\left(-\frac{1}{4}\right)$ $\approx -.25$ rad

In the terms of the sine function and its inverse, we have the following properties:

"undo"

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

restrictions

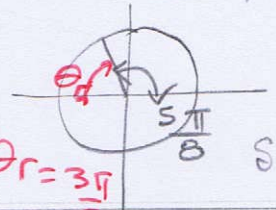
Find the exact value of composite functions:

$sin^{-1}(\sin\frac{\pi}{8}) = \frac{\pi}{8}$

$1.18 \approx sin^{-1}(\sin\frac{5\pi}{8}) = \frac{5\pi}{8}$?

Check w/calc ≠ NO

$\frac{5\pi}{8} \approx 1.96 \neq 1.18$



$\theta_r = \frac{3\pi}{8}$

$\sin^{-1}(\sin\frac{5\pi}{8}) =$

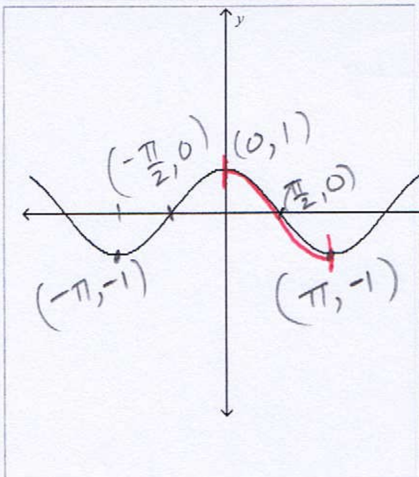
$\sin^{-1}(\sin\frac{3\pi}{8}) = 1.18$

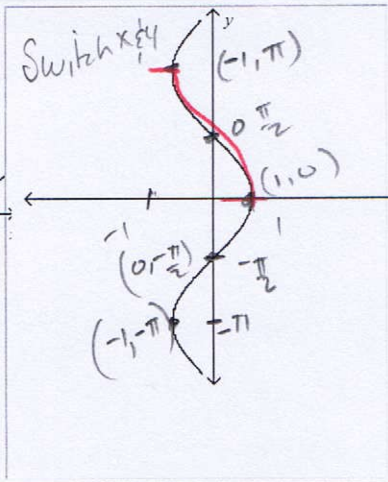
$\boxed{= \frac{3\pi}{8}} \approx 1.18$

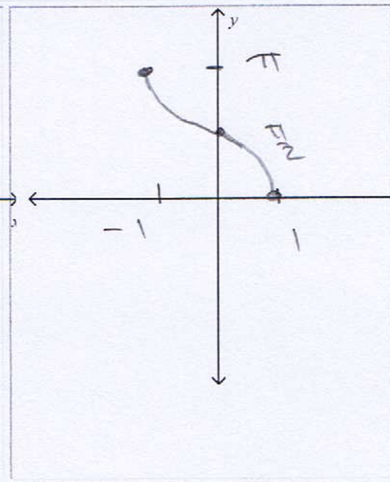
Calc. Check

$\theta = \frac{5\pi}{8}$ & θ_r in same Quadrant.

Inverse Cosine Function: cos^{-1} *also called* ***arrcosine*** *and written as* ***arrcsos***



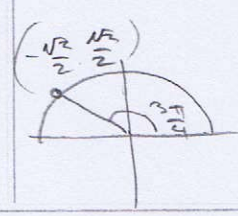




$\cos x = y$	$\cos^{-1} x = y$	$\cos^{-1} x = y$
Domain: $(-\infty, \infty)$	D $[-1, 1]$	Domain: $[-1, 1]$
Range: $[-1, 1]$	R $(-\infty, \infty)$	Range: $[0, \pi]$

By definition the inverse cosine function is the function cos^{-1} with domain of $[-1, 1]$ and range of $[0, \pi]$ defined by

$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y$$
$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad 0 \le y \le \pi$$

$\cos^{-1} 0$	$\cos^{-1} -\frac{\sqrt{2}}{2}$
$\cos\theta = 0$ $\theta = \frac{\pi}{2}$	$\cos\theta = -\frac{\sqrt{2}}{2}$ $\theta = \frac{3\pi}{4}$ 

In the terms of the cosine function and its inverse, we have the following properties:

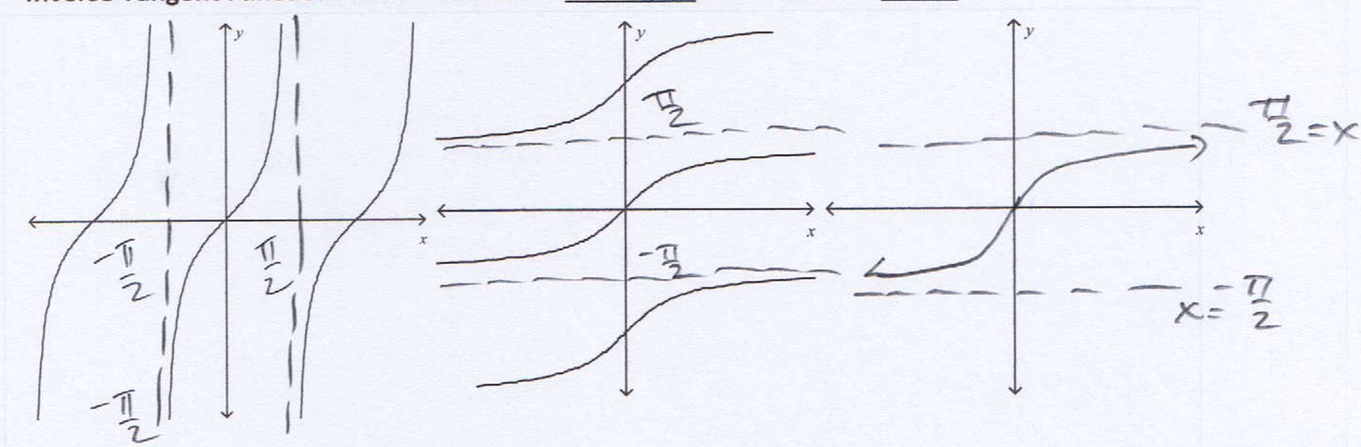
$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \qquad \text{where } 0 \le x \le \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

even function Trig identity
$\cos -x = \cos x$

$cos^{-1}\left(\cos\left(\frac{\pi}{12}\right)\right)$ $\cos^{-1}(\quad)$ Bounds $[0, \pi]$ $= \frac{\pi}{12}$	$cos^{-1}[\cos\left(-\frac{2\pi}{3}\right)]$ $\cos^{-1}(\quad)$ $[0\ \pi]$ not in Bounds cosine even: $\cos -\frac{2\pi}{3} = \cos\frac{2\pi}{3}$ ∴ $= \cos^{-1}(\cos\frac{2\pi}{3}) = \frac{2\pi}{3}$
$\cos(cos^{-1}\pi)$ $\cos^{-1}\pi = \theta$ $\cos\theta = \pi$ UnDEF with $\cos\theta$ the maximum output is 1	$\cos(cos^{-1}(-0.4))$ $= -.4$

minimum output is -1

Inverse Tangent Function: $\boldsymbol{tan^{-1}}$ *also called* *arctangent* *and written as* *arctan*



$tan\, x = y$

Domain: $(-\infty, \infty)$ except odd multiples $\pi/2$ → D:

Range: $(-\infty, \infty)$ → R:

$\tan^{-1} x = y$

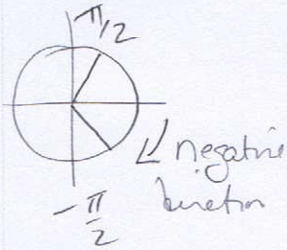
Domain: $(-\infty, \infty)$

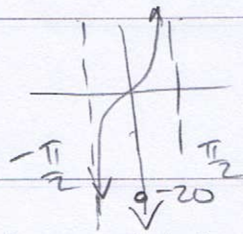
Range: $\left(-\frac{\pi}{2}\quad \frac{\pi}{2}\right)$

The inverse tangent function is the function tan^{-1} with domain of all real numbers and range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ defined by

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$
$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

EXAMPLE Evaluate the inverse tangent functions   Boundary $\tan\theta$ $\left(-\frac{\pi}{2}\ \frac{\pi}{2}\right)$

$y = \tan^{-1} 1$	$y = \tan^{-1} -\sqrt{3}$
$\tan ? = 1$ $\tan\theta = 1$ $\theta = \frac{\pi}{4}$	 $\tan\theta = -\sqrt{3}$ $\theta = -\frac{\pi}{3}$



$y = \tan^{-1}(-20)$ ← put into calc.

$\tan\theta = -20$ $\theta \approx 1.52$

In the terms of the tangent function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where } -\infty < x < \infty$$