Precalculus

Lesson 7.1: The Inverse Sine, Cosine and Tangent Functions

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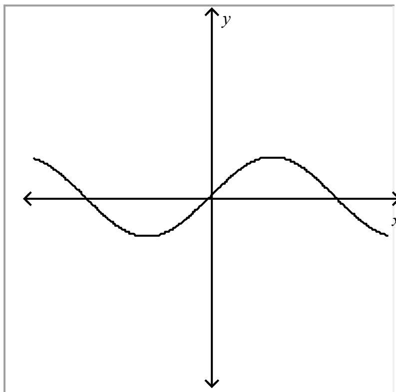
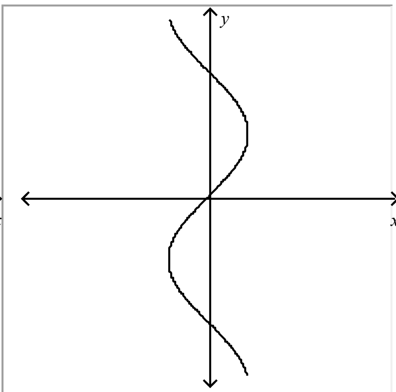
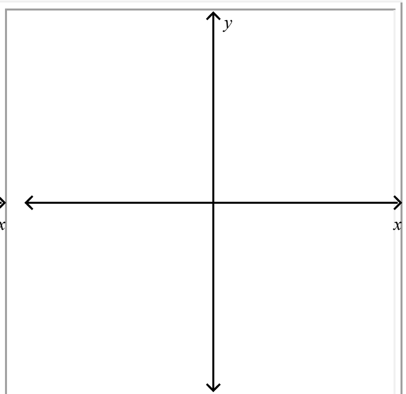
in Section 5.2 we discussed inverse functions. If a function is one-to-one if has an inverse. We are able to restrict the domain of a function to make it one-to-one. While trig functions are of course functions, they are not 1 – 1, so they do not have inverses. We can, however, force our trig functions into being 1 – 1 by limiting their domain.

Inverse Sine Function: $\boldsymbol{sin^{-1}}$ is *also known as* *arcsine* *and written as* *arcsin*

		
$\sin x = y$ Domain: Range:	$\sin^{-1} x = y$	$\sin^{-1} x = y$ Domain: Range:

By definition: the inverse sine function is the function sin^{-1}, with a domain of $[-1, 1]$ and range of $[-\frac{\pi}{2}, \frac{\pi}{2}]$ defined by:

$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$
$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

Finding the exact value of an inverse sine function:

$sin^{-1} 1$	$sin^{-1} -\frac{1}{2}$	$sin^{-1} \frac{3}{2}$

Approximate values of inverse sine functions may be found using a calculator. Express the answer in radians rounded to 2 decimal places.

$sin^{-1}\frac{1}{3}$	$sin^{-1}\left(-\frac{1}{4}\right)$

In the terms of the sine function and its inverse, we have the following properties:

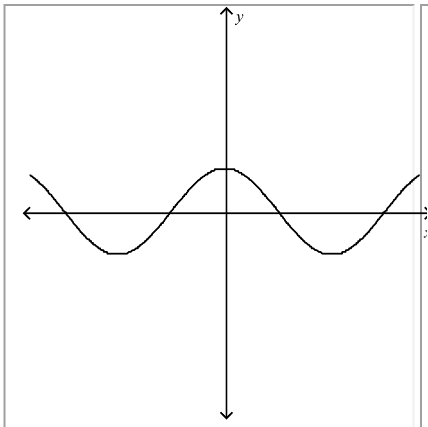
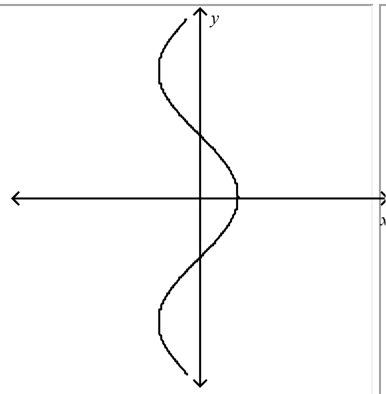
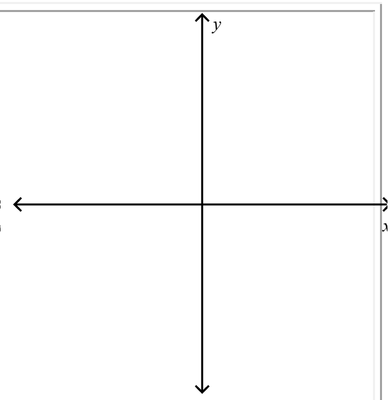
$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \leq x \leq 1$$

Find the exact value of composite functions:

$sin^{-1}(\sin\frac{\pi}{8})$	$sin^{-1}(\sin\frac{5\pi}{8})$

Inverse Cosine Function: $\boldsymbol{cos^{-1}}$ *also called* *<u>arrcosine</u>* *and written as* *<u>arccsos</u>*

		
$cos\ x = y$ Domain: Range:	$\cos^{-1} x = y$	$\cos^{-1} x = y$ Domain: Range:

By definition the inverse cosine function is the function cos^{-1} with domain of $[-1, 1]$ and range of $[0, \pi]$ defined by

$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y$$
$$\text{where} \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi$$

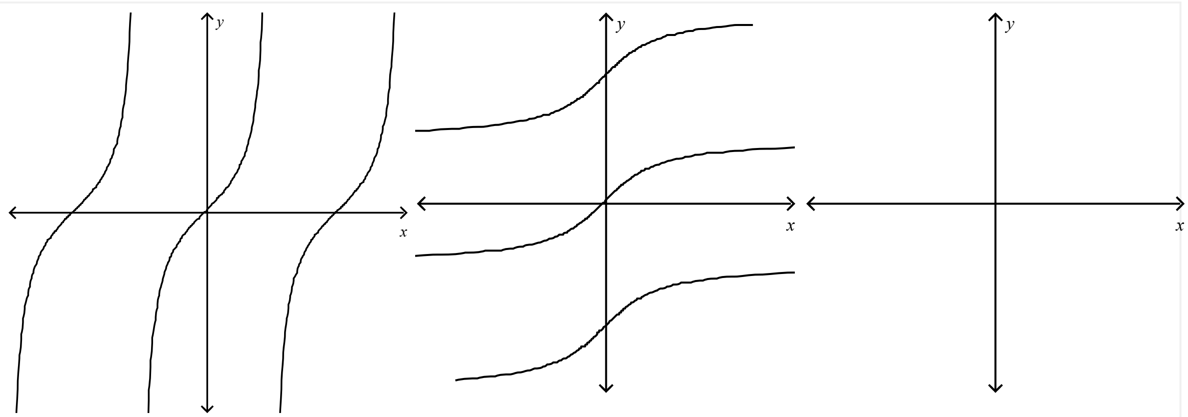
$\cos^{-1} 0$	$\cos^{-1} -\frac{\sqrt{2}}{2}$

In the terms of the cosine function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \qquad \text{where } 0 \leq x \leq \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \qquad \text{where } -1 \leq x \leq 1$$

$cos^{-1}\left(\cos\left(\frac{\pi}{12}\right)\right)$	$cos^{-1}[\cos\left(-\frac{2\pi}{3}\right)]$
$\cos(cos^{-1}\pi)$	$\cos\left(cos^{-1}(-0.4)\right)$

Inverse Tangent Function: $\boldsymbol{tan^{-1}}$*also called* *arctangent* *and written as* *arctan*



$tan\, x = y$

Domain:

Range:

$\tan^{-1} x = y$

Domain:

Range:

The inverse tangent function is the function tan^{-1} with domain of all real numbers and range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$defined by

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$

$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

EXAMPLE Evaluate the inverse tangent functions

$y = \tan^{-1} 1$	$y = \tan^{-1} -\sqrt{3}$
$y = \tan^{-1}(-20)$	

In the terms of the tangent function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where } -\infty < x < \infty$$