Precalculus
Lesson 14.4: Limits at Infinity
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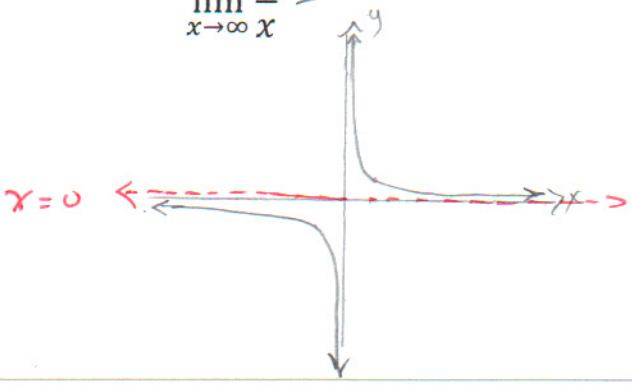
Just as the title of the lesson states, given a function $f(x)$ as x approaches infinity what happens to the value $f(x)$?

Find

$\lim_{x\to\infty} \frac{1}{x} = 0$ $\qquad$ $\lim_{x\to-\infty} \frac{1}{x} = 0$



L is the horizontal asymptote of $f(x)$

$\lim_{x\to\infty} f(x) = L$ & $\lim_{x\to-\infty} f(x) = L$

So we get a rule to remember:

If k is any positive integer, then:

$$\lim_{x\to\infty} \frac{1}{x^k} = 0 \qquad \textit{and} \qquad \lim_{x\to-\infty} \frac{1}{x^k} = 0$$

Evaluate:

$$\lim_{x\to\infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$$

* Multiply by "1" so that the variable gets into the $\frac{1}{x^k}$ form
* use highest degree of x
* use limit laws

$$= \lim_{x\to\infty} \frac{\frac{3x^2}{x^2} - \frac{x}{x^2} - \frac{2}{x^2}}{\frac{5x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x^2}}$$

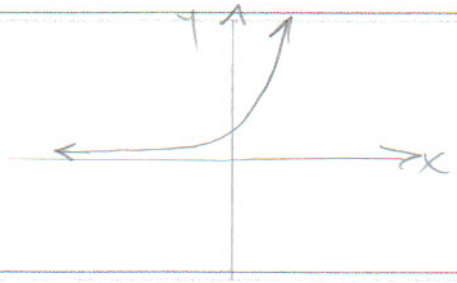
$$= \lim_{x\to\infty} \frac{3 - \frac{1}{x} - \frac{2}{x^2}}{5 - \frac{4}{x} + \frac{1}{x^2}} = \frac{3}{5}$$

($\frac{1}{x} \to 0$, $\frac{2}{x^2} \to 0$, $\frac{4}{x} \to 0$, $\frac{1}{x^2} \to 0$)

Finding a limit at negative infinity

$$\lim_{x\to-\infty} e^x = 0$$

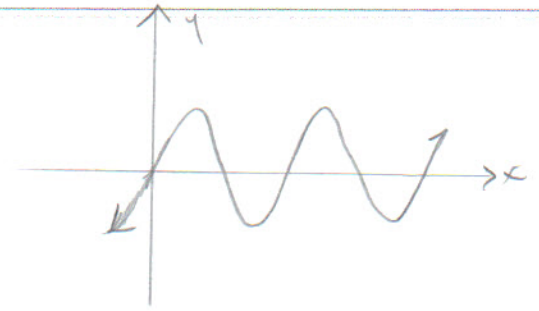
graphically



Can function at infinity have no limit??????

$$\lim_{x\to\infty} \sin x$$

oscillates: never goes to a single value



In chapter 12 we studied sequences: $a_1, a_2, a_3, \ldots a_n,$. Using limits we can determine the behavior of a sequence as n becomes large.

Convergent vs. divergent: Converge is when things come together from different directions so they eventually meet. Diverge is when things separate and go in different directions. Well, in sequences the term a_n may converge by approaching a number or it may not.....

Finding the Limit of a Sequence

$$\lim_{n\to\infty} \frac{n}{n+1} \cdot \left(\frac{\frac{1}{n}}{\frac{1}{n}}\right) = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} \to 0 \quad = 1$$

Multiply by "1"

Does the Sequence Converge or Diverge?

$a_n = (-1)^n \Rightarrow -1^1, -1^2, -1^3, -1^4, -1^5 \ldots$

$= -1, 1, -1, 1, -1 \ldots\ldots$

divergent cannot settle on a single value - bounces back & forth

Finding the Limit of a Sequence

Find the limit of the sequence given.

$$a_n = \frac{15}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{15}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{15}{6} \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{n^3}$$

$$= \frac{5}{2} \lim_{n\to\infty} \frac{n^1}{n}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= \frac{5}{2} \lim_{n\to\infty} \left(\frac{n}{n}+\frac{1}{n}\right)\left(\frac{2n}{n}+\frac{1}{n}\right)$$

$\frac{n}{n} \to 1$, $\frac{1}{n} \to 0$, $\frac{2n}{n} \to 2$, $\frac{1}{n} \to 0$

$$= \frac{5}{2}(2) = \frac{5}{1}$$