Precalculus

Lesson 14.2: Algebra Techniques for Finding Limits

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Limit Laws

$$\lim_{x\to a} f(x) \qquad \text{and} \qquad \lim_{x\to a} g(x)$$

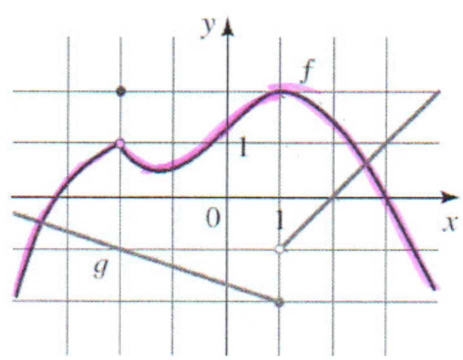
1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$
4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
6. $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$
7. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

Using the Limit Laws

Use the limit laws and the graphs of f and g in Figure to evaluate the following limits, if they exist.

$\lim_{x\to -2}[f(x) + 5g(x)]$

$\lim_{x\to -2} f(x) + 5\lim_{x\to -2} g(x) = 1 - 5 = -4$



$\lim_{x\to 1}[f(x)g(x)]$

$\lim_{x\to 1} f(x) \cdot \lim_{x\to 1} g(x) = \text{DNE}$

jump

$\lim_{x\to 2} \dfrac{f(x)}{g(x)}$

$\dfrac{\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)} = \dfrac{1.4}{0}$ DNE

$\lim_{x\to 1}[f(x)]^3$

$\left[\lim_{x\to 1} f(x)\right]^3 = (2)^3 = 8$

Some Special Limits

$$1.\lim_{x \to a} c = c$$

$$2.\lim_{x \to a} x = a$$

$$3.\lim_{x \to a} x^n = a^n$$

$$4.\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$$

Limits by Direct Substitution:

$$\lim_{x \to a} f(x) = f(a)$$

Using the Limit Laws: Evaluate the following limits.

$$\lim_{x \to 5} 2x^2 - 3x + 4$$

$$= 2\lim_{x \to 5} x^2 - 3\lim_{x \to 5} x + \lim_{x \to 5} 4$$

$$= 2(5)^2 - 3(5) + 4$$

$$50 - 15 + 4 = 49$$

$$\lim_{x \to -2} \frac{x^3 + 2x^2 + 1}{5 - 3x}$$

$$\frac{\lim_{x \to -2} x^3 + 2\lim_{x \to -2} x^2 + \lim_{x \to -2} 1}{\lim_{x \to -2} 5 - 3\lim_{x \to -2} x} = \frac{(-2)^3 + 2(-2)^2 + 1}{5 - 3(-2)}$$

$$= \frac{-8 + 8 + 1}{5 + 6} = 1/11$$

Finding Limits by Direct Substitution: Evaluate the following limits.

$$\lim_{x \to 3} 2x^3 - 10x - 8$$

yeah!

$$= 2(3^3) - 10(3) - 8$$

$$= 54 - 30 - 8$$

$$= 16$$

$$\lim_{x \to -1} \frac{x^2 + 5x}{x^4 + 2}$$

$$= \frac{(-1)^2 + 5(-1)}{(-1)^4 + 2} = \frac{1 - 5}{1 + 2} = -\frac{4}{3}$$

Finding a Limit by Canceling a Common Factor

$$\lim_{x\to 1}\frac{x-1}{x^2-1} = \lim_{x\to 1}\frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \lim_{x\to 1}\frac{1}{x+1} = \frac{1}{2}$$

* Solved graphically in Section 14.1 lesson.

Finding a Limit by Simplifying

$$\lim_{h\to 0}\frac{(3+h)^2-9}{h} = \lim_{h\to 0}\frac{\cancel{9}+6h+h^2-\cancel{9}}{h}$$

$h \to 0$ undefined!

$$= \lim_{h\to 0}\frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 6+h = 6$$

Never ever never!!! never... drop limit notation!

Finding a Limit by Rationalizing

×! conjugate!

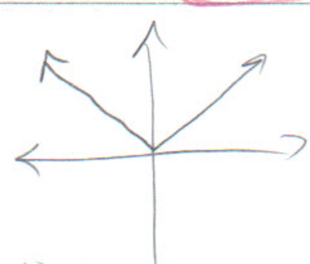
$$\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2} = \lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}\cdot\frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$$

$$\lim_{t\to 0}\frac{t^2+\cancel{9}-\cancel{9}}{t^2\cdot\sqrt{t^2+9}+3} = \lim_{t\to 0}\frac{\cancel{t^2}\ 1}{\cancel{t^2}\cdot\sqrt{t^2+9}+3}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = 1/6$$

Comparing Right and Left Limits: Show that

$$\lim_{x\to 0}|x| = 0 \Rightarrow |x| = \begin{cases} x & \text{for } x > 0 \\ -x & \text{for } x \geq 0 \end{cases}$$



$$\lim_{x\to 0}|x| = \lim_{x\to 0^+} x = 0$$

$$\lim_{x\to 0}|x| = \lim_{x\to 0^-} -x = 0$$

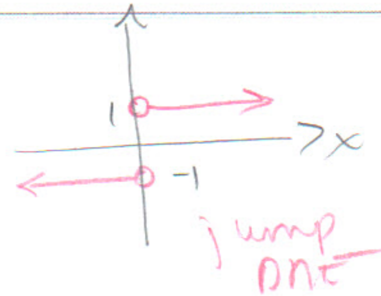
$\therefore \lim_{x\to 0}|x| = 0$

Prove that

$$\lim_{x \to 0} \frac{|x|}{x} = \text{DNE}$$

$|x| = x \quad \text{for } x \geq 0$

$|x| = -x \quad \text{for } x \leq 0$



$$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

x positive

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$$

RH limit ≠ LH limit

∴ DNE

The Limit of a Piecewise Defined Function

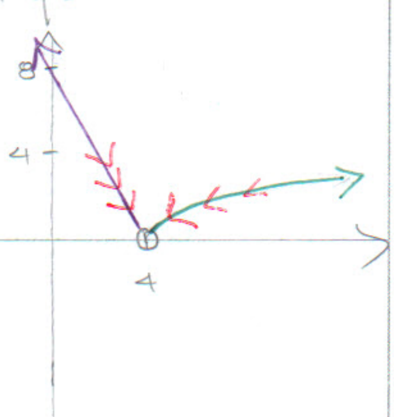
$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x < 4 \end{cases}$$

Piece 1 { Domain $x > 4$, graph $y = \sqrt{x-4}$ } $\sqrt{x}$ → Rt 4

Piece 2 { Domain $x < 4$, graph $y = 8 - 2x$ }

find:

$\lim_{x \to 4} f(x) = 0$



Using Limits:

$$\lim_{x \to 4} \sqrt{x-4} = 0$$

AND

$$\lim_{x \to 4} 8 - 2x = 0$$

$\lim_{x \to 4} f(x) = 0$